PAK Study Manual

Enterprise Risk Management (ERM) Exam

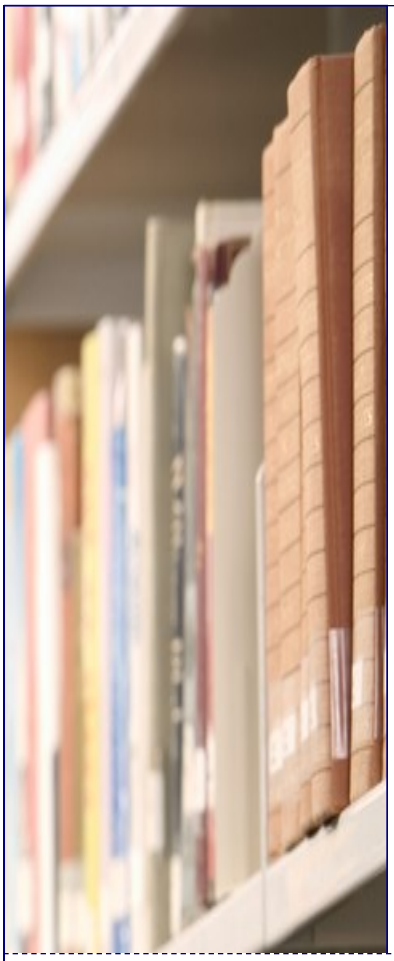
Spring 2021 Edition

PAK Study Manual for ERM Spring 2021

PRODUCT FEATURES

Features	PAK Study Manual	PAK Exam Aid	PAK Flash Cards	PAK Test Aid	PAK Online Seminar	PAK Study Manual Basic Package	PAK Study Manual Premium Package
Summaries	X					X	X
Relevant Past Questions (List)	X					X	X
400+ Practice Questions	X					X	X
10 Mock Questions	X					X	X
Suggested Schedule	X					X	X
Online Tutorial Videos	X					X	X
Email Support	X					X	X
110 Mock Questions		X				X	X
900+ Practice Questions		X				X	X
Case Study Analysis		X				X	X
Case Study Practice Question Set		X				X	X
Past Questions (Sorted PDFs)		X				X	X
Electronic Flash Cards + Anki			X			X	X
Audio Flash Cards			X			X	X
Condensed Summary			X			X	X
Mock Exam				X			X
Online Video Seminar					X		X
Practice Questions					X		X
Bonus materials							X

***All products are in electronic format.**
****The printed copy of the PAK Study Manual and the PAK Flash Cards can be purchased separately.**



PAK STUDY MANUAL

A printed copy of the manual can be purchased separately.

1. Summary (ERM Core + ERM Extension)

The PAK Study Manual covers the entire Enterprise Risk Management (ERM) syllabus (***The ERM extension is included***). Not only does it give you the detailed explanations on conceptual, calculation, and exam materials, but it also fills in the gaps among the topics that are not covered in the source readings. It helps you better understand and master the confusing logics and difficult materials.

In addition, it links the similar topics across readings together and connects them to the syllabus so that you can see the whole picture of this exam.

2. Relevant Past ERM SOA Exam Questions (List)

For each reading, we compose a list of relevant past exam questions (if any) so that you can locate the questions quickly and practice them immediately. This saves your time on searching what materials are relevant to this exam.

3. 400+ Practice Questions

One key point to pass this exam is to "practice" (Practice makes perfect!). Due to this reason, I include many practice questions in each reading (400+ in total) to refresh the materials just learnt and to strengthen your knowledge. More practice will be available in the PAK Exam Aid.

4. 10 Mock Exam Questions

The mock exam questions mimic the same difficulty level of the real exam questions. 10 mock exam questions and solutions are included in the PAK Study Manual to challenge your understandings. More practice will be available in the PAK Exam Aid.

5. Suggested Study Schedule (Detailed)

The syllabus is huge. It is very easy to lose track on your study. A clearly defined study schedule and some useful tips are included to help you better manage your schedule.

6. Online Tutorial Videos

10 videos are included to cover the confusing and difficult topics.

7. Email Support

Get questions? Please send me an email .

"The study materials were very helpful in preparing me for the exam. Most importantly I was better able to apply the things that I learned to exam style questions. It was very organized and valuable." By Marc Roberts

Read the whole story

DO YOU KNOW?

The PAK Study Manual and related aids are updated EVERY exam sitting.

You will see the most updated materials, examples, and explanations to help you master the concepts and pass this exam in the first attempt.

PAK FLASH CARDS

A printed copy of the flash cards can be purchased separately.

1. Electronic Flash Cards

- ◊ Summarize the key points in organized format.
- ◊ Include pros/cons, definition/description, etc.
- ◊ Contain around 350+ front/back flash cards (or 700+ slides).
- ◊ Read them in your smart phone device, tablet device, and/or computer.
- ◊ PDF version is also available.

2. Audio Flash Cards + Anki

- ◊ You can load them to your smartphone device and listen to them anytime/ anywhere you want.
- ◊ An Anki version is available.

3. Electronic Condensed Summary

- ◊ Summarize the key points in outline format.
- ◊ Quickly refresh all the important topics in the readings.

PAK TEST AID

1. Mock Exam

- ◊ This set of mock exam is different from those mock questions available in the PAK Exam Aid. You can write down your answers and send them to me. I will give you detailed feedbacks on how to improve your exam score.

PAK EXAM AID

1. 110 Mock Exam Questions and Solutions
The mock exam questions mimic the same difficulty level of the real exam questions. 100 mock exam questions and solutions (with ***case-study-specific questions***) are included to challenge your understandings.

2. 900+ Practice Questions and Solutions (ERM Core + ERM Extension)
One key point to pass this exam is to "practice" (Practice makes perfect!). Due to this reason, I include many practice questions in each reading (900+ in total) to refresh the materials just learnt and to strengthen your knowledge. Please note that this practice question set is different from the practice question set in the manual.

3. Case Study Analysis
This set connects the case study materials to the study materials so that you can see the picture on how they can be tested.

4. Case Study Practice Questions Set
This set help students to better understand how to apply the knowledge into the case study.

5. Past SOA Exam Questions (from All FSA Tracks) Relevant to This Exam
This set not only includes the past exam questions from the ERM exam, but also includes the past exam questions from all the other FSA exam tracks (e.g. QFI, LP, IRM, etc). It helps you better understand how the materials were tested and gets you familiar with the SOA exam question style.

DO YOU KNOW?

You can find the most updated information about the PAK Study Manual and related aids under the "Announcement" section on the front page of the PAK website.

"The Mock questions, Mock exam and Exam Aid are also very beneficial to make sure you're abilities are up to par before the real exam." By Wes Smith

Read the whole story

RELEASE SCHEDULE

Features	PAK Study Manual	PAK Exam Aid	PAK Flash Cards	PAK Test Aid	PAK Online Seminar	PAK Study Manual Basic Package	PAK Study Manual Premium Package
Summaries	11/15 + 12/15					11/15 + 12/15	11/15 + 12/15
Relevant Past Questions (List)	12/15					12/15	12/15
400+ Practice Questions	12/15					12/15	12/15
10 Mock Questions	11/15					11/15	11/15
Suggested Schedule	11/15					11/15	11/15
Online Tutorial Videos	11/15					11/15	11/15
Email Support	Anytime					Anytime	Anytime
110 Mock Questions*		2/15				2/15	2/15
900+ Practice Questions		2/15				2/15	2/15
Case Study Analysis*		2/15				2/15	2/15
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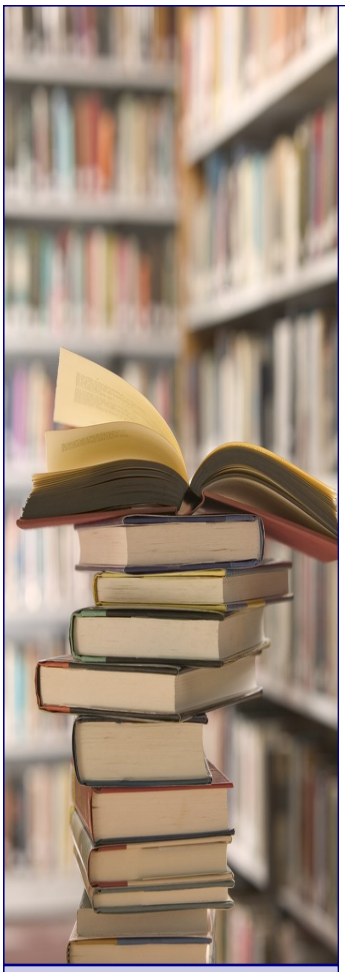
Condensed Summary			12/15			12/15	12/15
Mock Exam				3/15			3/15
Online Video Seminar					11/15 + 2/15		11/15 + 2/15
Practice Questions					2/15		2/15
Bonus materials							To Be Announced

* The release schedule of these items may be changed. It depends on when the SOA will release the new case study.



DO YOU KNOW?

If you are not sure which exam track to take, or how it can advance your career, you can send an email to Eddy and discuss your situation with him. He will share his work experience with you so that you can make your decision informatively.

PAK ONLINE SEMINAR

1. Over 70 video lectures to clarify and explain the key concepts/calculations in each syllabus reading (Core + Extension) and past exam questions
2. Analyze the new case study (e.g. how to answer the case-study-related questions)
3. Discuss the past exam questions (e.g. exam techniques, how to score)
4. Contain condensed outlines for each reading on the syllabus
5. Include practice questions (200+) to test your knowledge
6. Review the lectures and study at your own pace
7. On-demand video library compatible with smartphones (iPhone, Android, etc), tablets (iPad, Android), and PC/Mac devices.
8. Email support
9. Free access for 2nd attempt (only for those who scored 2-5)

PAK STUDY MANUAL PREMIUM PACKAGE

1. PAK Study Manual
2. PAK Exam Aid
3. PAK Flash Cards
4. PAK Test Aid
5. PAK Online Seminar
6. Bonus materials

SAMPLES?

You can find more samples on the PAK website.

IMPORTANT NOTES

1. Please note that all products (except flash cards) are in electronic (PDF) format. **No** hard copy is provided.
2. Once you make a purchase (please use your work email address), we will send you a confirmation email within 1 business days Once the files are available, we will send them to you through email. Please make sure that you put the correct email address when you purchase the PAK products. **If you do not receive the confirmation email, please send us an email (services@pakstudymanual.com).**
3. Please check your "junk" mailbox. Sometimes, our email is blocked.

MORE INFORMATION

Want more information? Please contact me at **eddy.chan@pakstudymanual.com** or visit **www.pakstudymanual.com**

COMMENTS FROM THE PAST CANDIDATES

You can find more comments from the past candidates here: PAK Testimonials.

WHERE TO PURCHASE PAK PRODUCTS

The PAK products are available at Actex, and Actuarial Bookstore.

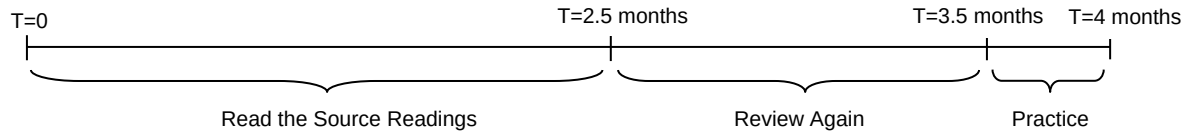
Frequent Answer Questions

Do You Need to Read the Source Readings?
Unlike the preliminary exams, reading the source readings (textbooks, SOA study notes, and online readings) is ***a must*** in the FSA exams. PAK Study Manual can help you understand the materials faster and memorize them quickly so that in the time-limited environment, you can be well-prepared for the exam.

How Much Time is Needed to Study for This Exam?
This varies by person. Usually it will take one 300-350 hours to study for a FSA exam. Please expect to spend the same amount of time for the ERM exam.

Study Schedule

From the date the SOA release the new syllabus to the exam date, there are around 4 months to study. How to plan your study schedule?



Read the Source Readings
Assume you take the ERM exam in this exam sitting. In general, it will take one 2 to 2.5 months to finish the whole syllabus. To study more efficiently, we highly suggest you following the steps below:

Step 1: Define Your Own Study Schedule
- Use the suggested study schedule as a reference
- Prepare your own study schedule *(Target 20-30 pages @weekday and 50-60 pages @weekend)*
- Expect to read the whole syllabus and the past exams 2 or 3 times before the exam

Step 2: Read the Source Readings Together with the PAK Study Manual
- Write down your notes in the study manual
- Highlight all the key points there
- Label any calculations that you will go over again later
- Go over the related past exam questions once you finish that reading

Step 3: Practice the SOA Past Papers
- Practice them once you finish your first-round of readings *(use the PAK Exam Aid)*
- Understand how the topics were tested and how the questions were answered

Review Again
After completing the three steps above, you probably have a general idea about how the exam looks like. Now you should review the source readings again but this time focus more on the key topics, clarify the confusing concepts/calculations, think of what can be tested and read them carefully (use my mock exam questions)

Practice
The last month is the most critical month. Here are the steps:
- Practice the past exams and my mock questions to identify what you still do not know
- Go back to the readings and find your answers *(or send us an email if you need help)*
- Start memorizing the key points *(use the PAK Flash Cards)*
- Use the PAK Test Aid to test your knowledge *(Send us your answers and we will give you detailed feedbacks on how to improve your score in the exams)*

More Information
We will explain how to prepare for this exam in much more details in the PAK Study Manual.

ERM Extension
You will need to choose one of the six ERM extensions in the ERM exam. ***If you do not know which one fits you, please feel free to send me an email***.

Questions?
We know you probably have a lot of questions in your mind regarding the exam or choosing study aids. Please feel free to contact us at eddy.chan@pakstudymanual.com or paulpeterson@pakstudymanual.com.

ERM-103-12: Developments in Modeling Risk Aggregation (p.72-89) (by Basel Committee on Banking Supervision)

Key Points

Key Points in This Reading
1) Understand different types of copulas.
2) Understand the properties of coherent risk measure.

Background

This reading discusses 3 risk aggregation methods. Let me summarize them here:

Comparisons among the Three Aggregation Methods

	Var-Covar approach	Distribution-based aggregation using copulas	Scenario-based aggregation
Description	○ It is a convenient and commonly used analytical technique that allows managers to combine marginal distributions of losses or distinct tail losses into a single aggregate loss distribution or tail loss estimate ○ The sole requirement is to characterize the level of interdependence of standalone losses, which is typically accomplished with a matrix of linear correlations ○ The lower the correlations on the non-diagonal elements of the matrix, the greater the level of diversification that can be realized with incremental (long) exposure to a risk component	○ A copula specifies the dependency structure among the individual random variables, and is used to join the marginal distributions (the distributions of each individual random factor) together ○ This decomposition of multivariate distributions into marginal distributions and a copula allows practitioners to match any set of individual distributions to a specified dependence structure ○ For instance, copulas having tail dependence can be applied to capture the observation that large losses from different risk types tend to strike simultaneously during stress situations	○ Building the scenarios requires digging deeply into the positions and identifying the risk drivers of these positions ○ Once the risk drivers are identified, the selected scenario can be simulated and risk exposures can be computed through specific algorithms and processes, leading to the derivation of the entire loss distributions under the considered scenarios
Strengths	○ It is simpler relative to other methods ○ It can be evaluated formulaically ○ It does not require fundamental information about the lower-level risks	○ Copulas allow direct control over the distributional and dependency assumptions used ○ The copula-based methods use entire loss distributions as inputs to the aggregation process ○ Copulas are usually easy to implement from a computational standpoint	○ It provides a consistent approach to aggregation ○ It forces the firm to undertake a deeper understanding of its risks ○ Its results can be interpreted easily
Weaknesses	○ It imposes a simple dependency structure that is not accurate in the real world ○ It cannot deal with the cases in which standalone risks are not actually exclusive but are integrated (e.g. market and credit risk)	○ Most of the copula methods are analytically complex and do not have closed-form solutions (no final number) ○ Its specification is difficult to interpret for non-experts ○ Fitting the parameters of a copula is a difficult statistical problem ○ Aggregating group-wide across the different risk types would require different copulas	○ Judgment and expertise are key to identifying risk drivers and deriving scenario sets ○ Simulation outcomes may be highly sensitive due to the uses of algorithms, processes, models and aggregation methods

You should also know the difference between correlation and dependency

Correlation: It quantifies a linear relationship (constant over time) between two random variables
Dependency: It quantifies a (linear or non-linear) relationship (vary over time) between two random variables
Correlation is a special case of dependency

Annex G: Technical Underpinnings of Aggregation Methods

Three Aggregation Techniques

1. Variance-covariance approach
2. Copula-based simulation
3. Scenario-based simulation

G.1 VarCovar approach

- It combines marginal distributions of losses into a single aggregate loss distribution (or tail loss estimate)
- The sole requirement is to characterize the level of interdependence of standalone losses (accomplished with a matrix of linear correlations)

Memorization: Advantages

The Main Advantages of VarCovar Approach

1. Use a limited number of inputs
2. Can be evaluated formulaically
3. Do not require fundamental information about lower-level risks

Statistical Foundation of VarCovar

- While VarCovar is a simple and highly tractable approach to risk aggregation, it fills in unspecified details about the nature of the loss distributions, which may or may not be accurate or intended (it assumes the lower-level risks are normally distributed)
- A statistical foundation of the variance-covariance approach is that the mean and variance of a real variable are known

Aggregated Capital Requirement

$$R = \lambda\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \operatorname{cov}(i,j)} = \sqrt{\sum_{i=1}^{N} w_i^2 r_i^2 + 2\sum_{i=1}^{N}\sum_{j=1,i\neq j}^{N} w_i w_j r_i r_j corr(i,j)}$$

R = The aggregated risk capital requirement
r = The lower-level risks which compose the aggregated risk (evaluated at a fixed confidence level)
$\operatorname{cov}(i,j)$ = The covariance between variables i and j
$\operatorname{corr}(i,j)$ = The correlation between variables i and j
$w(i)$ = Concentration weights for the lower-level risk sources
λ = The ratio of the tail risk value to the standard deviation (this is specific to the shape of the loss distribution and the choice of risk measure (eg, 99% VaR), but must be jointly applicable to both lower-level and aggregate risks)

Perfect linear dependence, independence and diversification

- The correlation matrix controls the level of diversification recognized by the firm

	Description
Perfect dependence	○ All values in the correlation matrix are 1 (equivalent to summing the lower-level risks to produce aggregate risk)
Independence	○ Applying the identity matrix (1s on the diagonal, 0s elsewhere) is equivalent to calculating aggregate risk as the square root of the sum of squared lower-level risks
Diversification	○ The lower the correlations on the non-diagonal elements of the matrix (diagonal elements must be equal to 1), the greater the level of diversification that can be realized with incremental (long) exposure to a risk component

The correlations within the VarCovar

- Practitioners usually interpret the elements of the correlation matrix as the linear correlations
- It provides only partial information about dependence if the distribution is not normal

Methods to (Try to) Overcome This Weakness

Method	Description
Stressed / tail correlation	○ Firms adjust the historical correlations based on expert judgment ○ The drawback is that it may be calibrated to match a desired overall outcome rather than receiving an appropriate level of independent justification
Rank correlation	○ Firms use rank correlation measures independently of assumed marginal distributions, possibly to accommodate a more conservative joint distribution or tail correlation matrix ○ Such a distribution can produce more severe and realistic examples of joint behavior under stress ○ But rank transformations do not preserve the assumptions required of VarCovar (uniform risk scaling)
Factor model	○ Firms use factor decomposition of lower-level risks to determine the correlation between them ○ Factor models estimate potential changes in the value of a risky asset based on its factor sensitivities to available risk factors and an idiosyncratic (residual) component ○ Factor structures can be estimated using regression and adjusted in specific ways to engineer a suitable correlation matrix

Non-Exclusive Risk

- VarCovar and other top-down aggregation tools (including copulas) also face difficulty in dealing with circumstances in which "standalone" risks are not actually exclusive but are believed to be integrated
    - E.g. the market risk and credit risk in a bank

Conclusions

- The constraints on distributions of the standalone risks (normal/Gaussian) is the limitation of VarCovar approach and it can lead to deeply misleading results

G.2 Distribution-based aggregation (Copula)

- The copula-based methods use entire loss distributions as inputs to the aggregation process
- These allow direct control over the distributional and dependency assumptions used
- But they are analytically complex

Memorization: Definition

Definition (Copula)

- The copula specifies the dependency structure among the individual random variables, and is used to join the marginal distributions together (to describe a multivariate distribution)

How Copulas are Used for Risk Aggregation

Steps for Aggregating Multiple Loss Distributions Using a Copula

1. Draw a joint sample of uniform random variables ($\tilde{u}_1,...,\tilde{u}_n$) from the distribution specified by the copula (Column 2 and 3 below)

2. Translate the sample from the copula distribution into a sample from the conjoined loss distribution by calculating the $\tilde{u}_1$-th percentile of X_1, the $\tilde{u}_2$-th percentile of X_2, etc ($F_{X_1}^{-1}(\tilde{u}_1),...,F_{X_n}^{-1}(\tilde{u}_n)$)
   (Column 4 and 5 below)

3. Calculate the realized sample for the aggregate loss as the sum of the percentiles drawn from each distribution ($F_{X_1}^{-1}(\tilde{u}_1)+...+F_{X_n}^{-1}(\tilde{u}_n)$)
   (Column 6 below)

4. Drawing many samples for the aggregate loss distribution will produce a simulated distribution and any measure of risk (such as VaR or expected shortfall) can be computed from this simulated distribution

Calculation Example
LN(mean = 2, STD = 1)
EXP(mean = 12)

Sample Number (1)	Copula Sample (first component) (2)	Copula Sample (second component) (3)	Lognormal Distribution Sample (4)	Exponential Distribution Sample (5)	Aggregate Loss Sample (6) = (5) + (4)
1	82.3%	40.6%	-2.9	-10.8	-13.7
2	50.3%	79.8%	-7.3	-2.7	-10.0

Lognormal Distribution Sample

$$F_{\ln} = \Phi\left(\frac{\ln X - \mu}{\sigma}\right)$$

$$\Phi^{-1}(F_{\ln}) = \frac{\ln X - \mu}{\sigma}$$

$$\sigma \times \Phi^{-1}(F_{\ln}) + \mu = \ln X$$

$$X = \exp(\sigma \times \Phi^{-1}(F_{\ln}) + \mu)$$

$$X = \exp(1 \times \Phi^{-1}(1 - 0.823) + 2) = 2.9$$

$$X = \exp(1 \times \Phi^{-1}(1 - 0.503) + 2) = 7.3$$

Exponential Distribution Sample

$$F_{\exp} = 1 - e^{-\lambda X}$$

$$1 - F_{\exp} = e^{-\lambda X}$$

$$\ln(1 - F_{\exp}) = -\lambda X$$

$$X = -\tfrac{1}{\lambda}\ln(1 - F_{\exp})$$

$$X = -\tfrac{1}{1/12}\ln(1 - (1 - 0.406)) = 10.8$$

$$X = -\tfrac{1}{1/12}\ln(1 - (1 - 0.798)) = 2.7$$

Distribution Functions of Copulas

- A copula can be specified completely by its distribution function
- Since all of the components of a copula range over the interval [0,1], a copula can be described as a function *C* mapping the Euclidean cube $[0,1]^n$ to the interval [0,1]
- This function must satisfy all of the conditions that a multivariate distribution function must satisfy (non-decreasing in each component, right continuity, limits of 0 and 1, rectangle inequality)
- Since all of the marginal distributions must be uniform, *C* must satisfy the condition that, for all arguments of the function and all *u* in the interval [0,1]: $C(1,...,1,u,1,...,1) = u$

Copulas from known distributions

- Given any multivariate distribution function $\mathbf{F}$ having marginal distribution functions $F_1,...,F_n$, the function: $C(u_1,...,u_n) = \mathbf{F}\left(F_1^{-1}(u_1),...,F_n^{-1}(u_n)\right)$ defines a copula

Memorization: Definition

Gaussian Copula

$C(u_1,...,u_n) = \mathbf{\Phi}_{\mathbf{\Sigma}}\left(\Phi^{-1}(u_1),...,\Phi^{-1}(u_n)\right)$ *where Φ is the standardized (univariate) normal distribution function*

This copula is easy to simulate because the underlying multivariate normal distribution with correlation matrix Σ is easy to simulate

Archimedean Copulas

$C(u_1,...,u_n) = \varphi^{-1}\left(\varphi(u_1),...,\varphi(u_n)\right)$ where $\varphi : [0,1] \to (0,\infty)$

Examples of Archimedean Copulas

Gumbel copula: $\varphi(x) = (-\ln x)^{\alpha}$ for $\alpha \geq 1$

Clayton copula: $\varphi(x) = (x^{-\theta} - 1)/\theta$ for $\theta > 0$

Frank copula: $\varphi(x) = \ln\left[(e^{\alpha x} - 1)/(e^{\alpha} - 1)\right]$

Archimedean Copulas
- The Archimedean copulas' distribution functions can be described in closed form
    - But advanced techniques are need in order to simulate Archimedean copulas

- Archimedean copulas are highly symmetric
    - This symmetry limits the use of these copulas to aggregating risks that are uniform and interact in the same manner
    - They cannot be used to model asymmetric behavior, which is quite commonly observed within risks

Measures of Dependence for Copulas

- Under the copula approach, the entire dependence structure is encapsulated in the choice of copula so any desired dependence structure can be specified

Relation between size and dependence
- The magnitude of a copula distribution function serves as an indicator of the level of dependence

Upper and Lower Bounds

Fréchet lower bound: $C(u_1,u_2)=\max(0,u_1+u_2-1)$

Between lower and upper bounds: $C(u_1,u_2)=u_1u_2$

Fréchet upper bound: $C(u_1,u_2)=\min(u_1,u_2)$

Correlations
- When a set of distributions is joined by a copula, the standard (Pearson) correlation matrix of the resulting multivariate distribution will vary with the marginal distributions
- In order to avoid having a model's results rendered invalid by the effect of the marginal distributions on the standard correlation, it is necessary to use measures of correlation that depend only on the copula itself
- This need is met by measures of rank correlation (Spearman rho and Kendall tau correlation coefficients)
- They are invariant under increasing functions, because they depend only on the relative rank of an observation within a data set rather than the actual value of the observation
- This implies that the measures will be the same for all multivariate distributions having the same copula, and that they can be calculated directly from the copula distribution function

Tail dependence
- Firms wish to capture the tail dependence in their copula models

Lower Tail Dependence

The coefficient of lower tail dependence for the copula: $\lim_{v\to 0}\frac{C(v,v)}{v}$

The copula exhibits lower tail dependence if this limit is greater than zero

Example (Proof)

Suppose we have a Clayton copula: $C(x_1,x_2)=(x_1^{-\theta}+x_2^{-\theta}-1)^{-1/\theta}$

$$\lim_{\mathbf{v\to 0}}\left(\frac{C(v,v)}{v}\right)=\lim_{\mathbf{v\to 0}}\left(\frac{(v^{-\theta}+v^{-\theta}-1)^{-1/\theta}}{v}\right)=\lim_{\mathbf{v\to 0}}\left(\frac{(2v^{-\theta}-1)^{-1/\theta}}{v}\right)=\lim_{\mathbf{v\to 0}}\left((2-v^{\theta})^{-1/\theta}\right)=2^{-1/\theta}$$

The lower tail dependence of a Clayton copula is $2^{-1/\theta}$

Gaussian Copula
- Its major drawback is that it does not exhibit any tail dependence between pairs of its variables

Multivariate (Student) t Copula
- It explicitly incorporates tail dependence into the aggregate risk distribution

Memorization: Comparison + Pros/Cons

Conclusions

Copula type:	Gaussian	t	Archimedean
Ease of simulation	Easy	Easy	Difficult
Capable of modeling tail dependence?	No	Yes	Yes
Symmetry	Symmetric in 2 dimensions, but generally asymmetric in higher dimensions		Standard construction is symmetric

Pros and Cons of Copulas

Advantages	Disadvantages
- They work directly with the percentile measures of the loss distributions (suited for aggregating risks)	- The specification of a copula is difficult to interpret (especially when the copula is given in terms of a distribution function rather than being derived from a known multivariate distribution)
- They are easy to implement from a computational standpoint	- Fitting the parameters of a copula is a difficult statistical problem (The estimators may change over time or during stress periods)

G.3 Scenario-Based Aggregation

- Scenario-based aggregation aggregates risk expressions to common underlying scenarios

Scenario-analysis - Determining risk drivers and exposures
- Profound knowledge is required to understand the risk positions, identify the risk drivers (internal/external), and develop the relevant scenarios
- Then, the firm uses scenario analysis to study the impact of the risk drivers and the changes in exposures
- It focuses on capturing and assessing potential "real-life" extreme events on the economic value of the financial institution

Scenario simulation
- Firms draw large numbers of random variables and process the random draws through models that describe particular processes to generate a large series of scenarios
- Firms may use different event generators to generate particular scenarios for different portfolios and then use VarCovar or Copula to combine the results
- Firms should determine the number of simulation runs required to obtain an adequate level of precision in the estimate

Three Types of Models Used
1. Models that describe and proxy "real physical processes or natural laws" (real-world models)
2. Models that describe processes for which there is no real physical model (risk-neutral models)
3. Models that combines the first two categories

Conclusions

- **Scenario-based aggregation aggregate exposures on the basis of common scenarios**
  - It requires a profound understanding of the risks the firm is exposed to
  - It relies heavily on a range of assumptions which have to be well understood and considered when interpreting the results
  - The results can be relatively easily and meaningfully interpreted in an economic and financial context

- **Scenario simulation requires sufficient computing power and solid IT programs and platforms**
  - The programs can allow certain scenarios to be given specific weights and additional scenarios can be (artificially) added to focus on particular aspects of the risk
  - These methods demonstrate a great deal of flexibility that does not exist in the more simple aggregation methods

Box A – Coherent Risk Measures

- The choice of a reference instrument (usually a one-year risk-free state bond) is a vital ingredient and can be considered as a yardstick to which the risk will be compared

Properties Required by a Coherent Risk Measure

Property	Formula	Description
Positive homogeneity	$p(\lambda X) = \lambda p(X)$	○ If a position has a risk, doubling the risk position (2*p(X)) leads to doubling the risk (p(2*X))
Sub-additivity	$p(X_1 + X_2) \leq p(X_1) + p(X_2)$	○ The risk of the sum of two positions ($p(X_1 + X_2)$) is always smaller or equal to the sum of the risks of the two positions ($p(X_1) + p(X_2)$)
Translation invariance	$p(X + \alpha) = p(X) - \alpha$	○ Adding to a portfolio an amount of cash invested in the reference instrument (α) reduces the risk measurement of this portfolio by the same amount
Monotonicity	$p(Y) \leq p(X)$ for $X \leq Y$	○ A position (Y) that always results in smaller losses than another position (X) always has a smaller risk (p(Y)) than the other position (p(X))

p(.) = Risk measure
X and Y = Variables (positions)
α = Constant (risk-free amount)

Examples of Coherent and Non-Coherent Risk Measures

	Description
Total exposure	○ It is a coherent risk measure ○ It is the most severe one for a given reference instrument
Standard deviation based risk measures	○ It is defined as the standard deviation relative to the expected value of the position ○ It is **not** a coherent risk measure as it does not fulfill the **monotonicity** property
Value at Risk	○ It reflects the quantile at a particular defined quantile level (α) ○ It is **not** a coherent risk measure as it does not fulfill the **sub-additivity** property
Expected Shortfall (or Tail-Value-at-Risk / CTE)	○ It is defined as the average value of the losses at quantiles lower than the specified quantile (α) ○ It is a coherent risk measure

Note
ERM-105-12 discusses the concepts and the calculations of coherent risk measures in details. ERM-103-12 just gives you a brief introduction of what they are.

Coherent measures of risk in practice

- When distributions are normal, Value at Risk and Expected Shortfall are quite close and behave similarly
- However, as soon as a risk position is characterized by a long tail behavior, the similarity between VaR and ES does not hold anymore

Calculation Example

Scenarios	1	2	3	4	5	6	7	8	9	10	11
Portfolio X	15	24	12	-12	11	-23	-7	22	-13	-2	-3
Portfolio Y	-5	26	34	-3	17	-7	33	12	-26	8	4
Portfolio X+Y	10	50	46	-15	28	-30	26	34	-39	6	1

Rearrange the numbers:

Smallest ←→ Largest

Portfolio X	-23	**-13**	-12	-7	-3	-2	11	12	15	22	24
Portfolio Y	-26	**-7**	-5	-3	4	8	12	17	26	33	34
Portfolio X+Y	-39	**-30**	-15	1	6	10	26	28	34	46	50

VaR(85%) for portfolio X = 13
VaR(85%) for portfolio Y = 7
VaR(85%) for portfolio X+Y = 30

Subadditivity: $VaR(X+Y) \leq VaR(X) + VaR(Y)$
But in this case: $30 \not\leq 13 + 7 = 20$ → This implies that aggregating the portfolio increases the total risk
→ It does not make sense

VaR is subadditive only if the distribution of the variable is normally distributed

Coherent risk measure and use of scenarios

- The representation theorem establishes a link between coherent risk measures and scenarios
- It states that a coherent risk measure is fully defined by a family of generalized scenarios and vice versa
- This property emphasizes and favors the use of scenarios by financial institutions to assess their risks as it allows more than the other methods to stay compatible with the coherence of the risk measure which has be shown to be a fundamental property

Box B – Correlations vs dependencies

Brief Explanations

Correlation: It quantifies a linear relationship (constant over time) between two random variables

Dependency: It quantifies a (linear or non-linear) relationship (vary over time) between two random variables
Correlation is a special case of dependency

Dependence vs. Correlation

	Dependence	Correlation
Independence	In statistics, Event A and Event B are *independent* if Pr (A | B) = Pr (A) or, equivalently, Pr (B | A) = Pr (B) Verbally: "Probability of A given B is equal to the Probability of A" "Probability of A is the same whether or not B occurs"	
Dependence vs. Correlation	Dependence means that the probability distribution of a variable is different depending on the state of the other variable	Correlation is a commonly used label for specific measures of dependence between pairs of variables
Characterization & Scaling	Qualitatively, there are varying degrees of dependence Variables that are highly dependent may have conditional distributions (probability of A given B) that are very different from their unconditional distributions (probability of A assuming nothing about B) The degree of dependence may vary with the value of the conditioning ("given") variable If extreme values for that variable are associated with relatively high conditional probabilities for dependent variables, which may signify high tail dependence	Correlation is a normalized measure of dependence "Correlation coefficient" refers to Pearson Product Moment Correlation This is a measure of linear relationship and is scaled from -1 to 1 Other correlation coefficients include Spearman's Rank Correlation & Kendall's Tau, also scaled from -1 to 1 Independent random variables have a correlation of zero The closer to -1 or 1, the stronger the level of dependence
Limits	Dependence is described in full only by knowing the entire joint probability distribution	Correlation measures can be limited in how they represent dependence A set of variables can have important dependencies (at the tail) that are not represented clearly by a specific measure of correlation Also, a given correlation might not distinguish between two very different joint distributions
	Correlation measures often prove to be unstable during stressful periods It can be difficult or impossible for risk managers to obtain reliable, time-independent measures of dependence due to potential changes in the overall dependence structure	

SOA ERM Past Exam Questions Related To This Reading

SOA ERM Core Fall 2017 Q4c (Must Read)
SOA ERM Core Spring 2017 Q1b (Must Read)
SOA ERM Core Fall 2015 Q6 (Must Read)
SOA ERM Core Spring 2015 Q5d (Must Read)
SOA ERM Core Fall 2014 Q7a,b (Must Read)
SOA ERM Core Fall 2013 Q1 (Relevant)
SOA ERM Core Fall 2012 Q3d(iii)-(iv) (Relevant)

PAK Study Manual (Practice Questions) for ERM Spring 2021

(Sample)

Note

1. 400+ Practice Questions/Solutions are included in the PAK Study Manual.

Textbook Reading: Value at Risk Ch.12

Q1: Securities firms commonly use simulation techniques (known as Monte Carlo methods) to value complex derivatives and to measure risk. Simulation methods approximate the behavior of financial prices by using computer simulations to generate random price paths. Compare the pros and cons of using Monte Carlo simulation.

Q2: Suppose each interval is one day. The initial stock price is $60. The instantaneous drift rate is 0.6% and the instantaneous volatility is 3%. Assume we generate two standard normal random variable: the first random variable is -0.80, and the second one is 0.90. The first random variable will be used to calculate the ending stock price at day 1. The second variable will be used to calculate the ending stock price at day 2. Calculate the ending stock price at day 2.

Q3: Bootstrap method is used to generate random numbers by sampling from historical data (empirical distribution. Compare the pros and cons of using bootstrap method.

$$\text{Covariance Matrix} = \begin{bmatrix} 0.04 & 0.16 & 0.10 \\ 0.16 & 0.73 & 0.46 \\ 0.10 & 0.46 & 0.54 \end{bmatrix}$$

Q4: Suppose we have a covariance matrix R above. It can be decomposed into its Cholesky factors: $R = TT'$, where T is a lower triangular matrix with zeros in the upper right corners. Calculate the T.

Q5-Others are not shown in this sample.

Textbook Reading: Value at Risk Ch.12

S1:

Pros and Cons of Using Monte Carlo simulation

Advantages	Disadvantages
1. It is the most powerful approach to VAR due to its flexibility	1. This approach involves costly investments in intellectual and systems development
2. It accounts for a wide range of risks and complex interactions	2. It requires substantially more computing power than simpler methods
3. It accounts for nonlinear exposures and complex pricing patterns	
4. Simulations can be extended to longer horizons	
5. It can be used for operational risk measurement, and integrated risk management	

S2:

Step *i*	Initial Price S_{t+i-1}	Random Variable ε_i	Increment $\Delta S = S_t\left(\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right)$	Current Price $S_{t+i} = S_{t+i-1} + \Delta S_{t+i-1}$
1	60.00	-0.80	-1.08	58.92
2	58.92	0.90	1.94	60.86

$$\Delta S_t = S_t\left(\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right) \qquad S_{t+1} = S_t + \Delta S_t$$

$$\Delta S_1 = 60\times\left(0.006\times 1 + 0.03\times(-0.80)\times\sqrt{1}\right) = -1.08 \qquad S_1 = 60 + -1.08 = 58.92$$

$$\Delta S_2 = 58.92\times\left(0.006\times 1 + 0.03\times(0.90)\times\sqrt{1}\right) = 1.94 \qquad S_2 = 58.92 + 1.94 = 60.86$$

S3:

Pros and Cons of Using Bootstrap Method

Advantages	Disadvantages
1. It can include fat tails jumps, or any departure from the normal distribution	1. For small sample sizes *M,* the bootstrapped distribution may be a poor approximation of the actual one so sufficient data points are needed
2. It accounts for correlations across series because one draw consists of the simultaneous returns for *N* series	2. It relies heavily on the assumption that returns are independent so by resampling at random, any pattern of time variation is broken

Q4-Others are not shown in this sample.