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4 HOURS

Exam 9

Financial Risk and Rate of Return

INSTRUCTIONS TO CANDIDATES

1. This 54.5 point examination consists of 18 problem and essay questions.
2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.
 - Write your Candidate ID number and the examination number, 9, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
 - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – DO NOT WRITE ON THE BACK OF THE PAPER. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as “Page 1 of 2” on the first sheet of paper and then “Page 2 of 2” on the second sheet of paper.
 - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
 - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

END OF INSTRUCTIONS

4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
 - Verify that the table of the Normal Distribution is attached to the examination after the last question.
5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.
7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper. Candidates may obtain a copy of the examination from the CAS Web Site. All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 28, 2012.

1. (3 points)

Given the following information:

Asset	Expected Return	Standard Deviation
A	12%	> 0%
B	10%	> 0%
C	6%	> 0%
D	3%	= 0%

- The coefficient of risk aversion is 3.
- The weight given to Asset A in the optimal risky portfolio is 40%.
- The standard deviation of the optimal risky portfolio is 16%.
- The slope of the capital allocation line is 0.35.

a. (2.5 points)

Construct an optimal investment plan using some of each asset (A, B, C, and D) and justify your proposed plan. Include brief descriptions of the proportion to be invested in each asset and the overall expected risk and return results.

b. (0.5 point)

Explain why the proportion invested in Asset D may be the only difference between plans for investors with the same assets.

2. (2 points)

Given the following information for \$1,000 par-value zero coupon bonds:

Maturity (years)	Price
1	\$952.38
2	\$902.73
3	\$845.55
4	\$807.23

Year	Forward Rate
1	5.0%
2	5.5%
3	5.5%
4	6.0%

- Interest rates are compounded annually.

Design an arbitrage strategy and calculate the resultant profit.

3. (3 points)

An insurance company's current investment manager has been using fundamental analysis to select stocks for the company's equity portfolio since 2009. The company has recently hired a new CFO who is a firm believer in market efficiency.

a. (1.5 points)

Briefly describe three forms of the efficient-market hypothesis and assess the relevance of each one to the CFO's evaluation of the current investment manager's strategy.

b. (1.5 points)

The following table compares the rates of return on the company's portfolio to the returns on the S&P 500:

Rates of Return		
Year	Portfolio	S&P 500
2009	+26.7%	+26.5%
2010	+15.5%	+15.1%
2011	-8.5%	-9.0%

The investment manager argues that the portfolio performance is strong evidence against the efficient-market hypothesis. Present a rebuttal that includes three reasons why that position may not be valid.

4. (1.75 points)

Given the following information to determine the interest rate sensitivity of a company's loss reserves for accident year 2013:

- Projected loss payout pattern:

Development Year	2013	2014	2015	2016
Percentage Paid	30%	35%	25%	10%

- Losses are paid at the end of each development year.
- Inflation rate is projected to be 0% for the next four years.
- The risk-free rate is 2%, compounded annually.
- The annual yield on company's investable assets is 4%, compounded annually.
- Ultimate losses for accident year 2013 are expected to be \$5 million.

a. (0.75 point)

Calculate the Macaulay duration of loss reserves for accident year 2013.

b. (1 point)

Illustrate to the management team how to immunize the loss reserve liability of accident year 2013 from interest rate fluctuations and briefly describe two limitations associated with the proposal.

5. (3.75 points)

Given the following information for a company's new line of insurance:

- An annual premium of \$3,115 is collected at the beginning of the year.
- Losses of \$2,100 are paid at the end of the year.
- Expenses of \$1,050 are paid at the beginning of the year.
- The company's surplus is \$1,400.
- The rate of return on surplus is 15%, compounded annually.
- The risk-free interest rate is 7%, compounded annually.
- The company expects to retain 85% of its policyholders from one year to the next.
- The company's current pricing strategy is that pricing does not vary with the current interest rate.
- The Duration of Current Economic Value = 1.0.

Explain how to use pricing strategy, as described by Panning in "Managing Interest Rate Risk: ALM, Franchise Value, and Strategy", to reduce the company's duration of total economic value and calculate the reduction in duration.

6. (1.5 points)

Company A has a \$10 million bond that pays 6% annually, with coupons payable every six months. Current six-month LIBOR rates are 2.5%. Company A expects LIBOR rates to rise at a steady rate of 15 basis points semi-annually over the next three years.

Company B offers an interest-rate swap for the Company A bond at LIBOR plus 25 basis points over the next three years.

Calculate the expected cash flows from this swap to Company A over the next three years if interest rates rise at Company A's expected rate.

7. (4 points)

Given the following information for a multi-national insurance company:

Operating Country	Written Premium at Year-end (000,000's)		Local Currency Holdings as a Percent of Local Liabilities at Year-end 2012
	2010	2012	
United States	\$10	\$40	2%
Canada	\$15	\$25	50%
Mexico	\$5	\$20	5%

- Quarterly, the Chief Risk Officer of the company uses the most recent balance sheet and assumptions of normal economic conditions to set intraday and interday liquidity risk thresholds.
- Aside from the local currencies, the balance of the insurance group's assets is in real estate and long-term sovereign debt obligations.
- Most of the Canadian premiums are life insurance while the U.S. premiums are mostly property with heavy concentrations in catastrophe-prone areas.
- The group has three different reinsurance treaties, one for each geo-political operating country, purchased from the same direct reinsurer.
- The group maintains a strong relationship with a global capital market player.

Describe four liquidity risks for this insurance company and what the CRO should consider in addressing these risks.

8. (5 points)

An analyst is managing a portfolio of high-risk subprime mortgages.

a. (3 points)

Describe how the analyst can construct AAA-rated securities out of this portfolio and how they would function. Include assumptions made in order for the securities to be rated at AAA.

b. (1.25 points)

Assess the role and impact of structured financial vehicles on the financial crisis of 2007-2008.

c. (0.75 point)

Defend or dispute the following statement:

"Credit rating agencies are to blame for the erosion of structured finance surrounding the events of 2007-2008."

9. (2.5 points)

a. (0.75 point)

Briefly describe three features of Value at Risk that make it a popular statistic for measuring exposure to market risk.

b. (1.75 points)

An actuary at an insurance company wants to use Value at Risk to allocate capital by line of business.

Discuss a process to do this, including three issues that the actuary should consider, and a motivation and a consideration in selecting an appropriate risk horizon.

10. (2 points)

An insurer estimates its 99% Value at Risk metrics for its three risk sources as follows:

Risk Source	99% VaR
Underwriting Risk	\$100 million
Reserve Risk	\$80 million
Credit Risk	\$70 million

For combinations of these sources, estimates of the 99% Value at Risk metrics are as follows:

Risk Sources	99% VaR
Underwriting and Reserve Risk	\$120 million
Underwriting and Credit Risk	\$130 million
Reserve and Credit Risk	\$140 million
Underwriting, Reserve, and Credit Risk	\$180 million

The insurer decides to use the 99% Value at Risk metric as an estimate of the risk capital.

a. (0.5 point)

Calculate the portion of total risk capital allocated to underwriting risk using the proportional method.

b. (0.5 point)

Calculate the portion of total risk capital allocated to reserve risk using the incremental method.

c. (1 point)

Decide whether the insurer should allocate its total estimated risk capital or its actual book value to the risk sources. Give two reasons for your decision.

11. (2.5 points)

Given the following information about an insurance company that writes two lines of business:

	Premium	Loss & LAE Reserves	Beta of Liabilities
Line A	\$500 million	\$600 million	0.25
Line B	\$500 million	\$200 million	0.35

- The company has \$1 billion of surplus and no liabilities other than loss and LAE reserves.
- The company's assets have a beta of 0.50.
- The risk-free rate of return is 4%.
- The equity market rate of return is 10%.

a. (0.5 point)

Calculate the expected rate of return on the firm's equity.

b. (1 point)

Calculate the combined ratios of the two lines of business.

c. (1 point)

Describe two potential problems with using CAPM to set profitability targets for the lines of business.

12. (1.5 points)

Below is a formula for the return on equity.

$$T/S = I/A (1 + R/S) + U/P \times P/S$$

Where:

T = Total after-tax return to the insurer

I = Investment gain or loss (after tax)

U = Underwriting profit or loss (after tax)

P = Premium income

A = Total assets

R = Reserves and other liabilities (excluding equity in unearned premium reserves)

S = Stockholder's equity (capital, surplus, and equity in unearned premium reserves)

a. (0.5 point)

Explain why when P/S increases, I/A will tend to decrease.

b. (0.5 point)

Explain why when P/S increases, U/P will tend to increase.

c. (0.5 point)

Explain why when U/P increases, I/A will tend to increase.

13. (4 points)

An insurance company writes two homeowners accounts with the following exposures to independent events:

Event	Probability	Account A Loss	Account B Loss	Total (A+B)
1	0.1%	200	100	300
2	1.0%	100	50	150
3	2.0%	80	60	140

Target return on equity = 15%

a. (2 points)

Calculate the renewal risk load for each account using the marginal surplus method, targeting a 2% probability of ruin.

b. (0.75 point)

Ten thousand policy years of losses are simulated using the data above.

Explain whether a company should or should not expect to achieve its target return on equity using the risk loads from part a. above.

c. (0.75 point)

Explain how the answer to part b., above would change if the marginal variance method of calculating risk loads were used.

d. (0.5 point)

The company uses the CAPM framework to set target return on equity.

Explain the impact on each account's renewal risk load under the marginal surplus method if the market risk premium increases.

14. (1.75 points)

In his discussion of Feldblum's "Risk Loads for Insurers," Bault compares approaches to risk load calculation based on ruin theory to those based on the CAPM framework. The following table summarizes total surplus allocation for these two approaches:

Approach	Surplus Allocated
Ruin Theory	$zS - R$
CAPM	zS

z = z-score at target probability

S = standard deviation of losses

R = expected return from operations

a. (0.75 point)

Explain how the difference in the surplus allocation between the two approaches relates to the traditional applications of ruin theory and CAPM.

b. (1 point)

Describe two factors that might make S insufficient as a measure of an insurance operation's variability. Assume past loss experience is used to estimate S .

15. (3.75 points)

An actuary has been asked to determine the risk load for a reinsurance contract that will make a single loss payment at the end of one year. If the contract is written, the reinsurer will invest the premium and any other allocated assets.

Given the following information:

Risk-free rate	2%
Yield rate of current investment portfolio	5%
Target return on equity	12%
Funds invested by reinsurer	\$4,000,000
Mean value of loss for the contract	\$3,000,000
Standard deviation of loss for the contract	\$540,000

a. (1 point)

Calculate the reinsurance premium using the swap method.

b. (0.75 point)

Describe what the loss safety level represents and what implications a higher loss safety level has for a reinsurer.

c. (2 points)

Suppose the assumption of a single loss payment is relaxed, allowing multiple loss payments. Design a simple process to determine a risk load using stochastic modeling techniques.

16. (3.5 points)

Companies A, B and C are competitors in a single insurance market. Each company has submitted a rate filing to state X for their proposed premium rates.

	Expected Loss & Expense	Proposed Rates	Premium-to-Capital Ratio
Company A	\$295	\$320	1.50
Company B	\$310	\$330	2.25
Company C	\$330	\$345	4.00

a. (1 point)

The local regulator claims that the rates proposed by Company C are excessive.

Defend the regulator's claim that Company C's rates are excessive.

b. (1 point)

Defend Company C's position that their rates are not excessive.

c. (1.5 points)

A consumer advocacy group argues that Company C's returns are excessive because the actuary did not account for all forms of investment income in the calculation of an adequate rate of return. The actuary contends that the calculations are correct.

Identify two types of investment income that may be excluded from the calculation and defend the decision to exclude them.

17. (4.5 points)

A property and casualty insurance company is doing a rate review for one of the lines of business that it writes. Given the following information for this line of business:

- Policies are in-force for one year.
- Premium is collected at the beginning of the policy period.
- A fixed expense amount of \$50 per policy is paid by the company at the beginning of the policy period.
- Variable expenses equal to 20% of the premium are paid by the company three months after the policy effective date.
- Losses averaging 35% of the policy premium are paid by the company six months after the policy effective date.
- Losses averaging 25% of the policy premium are paid by the company eighteen months after the policy effective date.
- The company allocates capital in a way that results in a premium-to-capital ratio of 2:1 for this line of business.
- The company's investment portfolio earns a 5% rate of return.
- Investment income is earned at the end of a given policy year.
- Supporting capital is released twenty-four months after the policy effective date.
- Only supporting capital is invested; investment income is not earned by all other assets.
- Ignore taxes.

a. (2.5 points)

Calculate the premium that the company must charge in order to earn a 10% internal rate of return (IRR) for this line of business.

b. (2 points)

Assume that regulators insist that the company charge premium rates 5% below what was calculated in Part A.

- i. Propose to upper management two modifications that the company could make to still earn the target 10% IRR for this line of business while charging no more than the regulated premium amount.
- ii. For each modification in part i. above, calculate the premium that the company must charge in order to earn at least a 10% IRR for this line of business using each modification.

18. (4.5 points)

Using the present value cash-flow return model, a new actuary determined the underwriting profit provision to be 5.5%.

Given the following for a single policy:

Loss	\$75
Fixed expenses	\$10
Variable expense ratio	24%
Tax rate	34%
Premium-to-surplus ratio	2.0
Equity-to-surplus ratio	1.3
Investment yield	10%
Rate used for discounting	8%
Target return	15%

Losses are paid according to the following pattern at the end of each quarter:

Quarter	Loss Payout Pattern
1	35%
2	25%
3	25%
4	15%

- Premium and expenses are paid at policy inception.
- Surplus equals investible equity and is assigned to the policy at inception and released at policy expiration.
- Equity is assigned to the policy at inception and released at policy expiration.

Demonstrate that the underwriting profit provision of 5.5% is consistent with the underwriting profit calculated by Present Value Cash Flow Return Model.

Tables of the Normal Distribution



Probability Content from $-\infty$ to Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Values of z for selected values of $\Pr(Z < z)$

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z < z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Exam 9
May 2012

Examiners' Report
with Sample Solutions

Spring 2012 Exam 9 - Model Solution & Examiner's Report

Question 1:

Part a:

Model Solution 1:

Asset D represents the risk free asset, since

$$\sigma = 0$$

$$\frac{E(r_P) - r_f}{\sigma_P} = .35 \text{ where } E(r_P) = \text{Exp return on optimal risky portf}$$

σ_P = std of optimal risky portfolio

$$\begin{aligned} E(r_P) &= (.35)\sigma_P + r_f \\ &= (.35).16 + .03 \\ &= .086 \end{aligned}$$

$$\begin{aligned} E(r_P) &= (.40)(.12) + w_B(.60)(.10) + (1 - w_B).60(.06) \\ .086 &= .048 + w_B.06 + (1 - w_B).036 \\ .038 &= w_B.024 + .036 \\ \Rightarrow w_B &= .0833 \end{aligned}$$

$$\begin{aligned} \Rightarrow w_B^* &= .0833 \times .60 = .05 \rightarrow 5\% \text{ weight to asset B} \\ \Rightarrow w_C^* &= 1 - .40 - .05 = .55 \rightarrow 55\% \text{ weight to asset C} \end{aligned}$$

std deviation of portfolio including the risk free asset is $y\sigma_P = \sigma_C$
where y is the percentage invested in the optimal risky portfolio

$$\text{let Utility function } \Rightarrow U = E(r_C) - .5A\sigma_C^2$$

where C represents portfolio including risk free asset

so y that maximizes U is

$$\begin{aligned} y^* &= \frac{E(r_P) - r_f}{3\sigma_P^2} \\ &= \frac{.086 - .03}{3(.16)^2} \\ &= .729 \end{aligned}$$

so .729 is invested in the risk free asset

Explanation:

[I took derivative of U w.r.t. to y and set it equal to zero to solve for y]

$$E(r_C) = yE(r_P) + (1 - y)E(r_D)$$

so

	Proportion invested	E(r)
Asset A	$(.729)(.40) = .292$.12
Asset B	$(.729)(.05) = .036$.10
Asset C	$(.729)(.55) = 0.401$.06
Asset D	.271	.03

$$\text{Overall risk} = \sigma_C = y\sigma_P = (.729)(.16) = .1166$$

$$\text{Overall return} = \sum \text{proportion invested (from above)} \times E(r) = .07083$$

Model Solution 2:

Optimal risky portfolio $w_A = 0.4$

$$\sigma_P = 0.16$$

$$0.4^2(\sigma_A^2) + w_B^2(\sigma_B^2) + (0.6 - w_B)^2(\sigma_C^2) = 0.16^2$$

$$E(r_P) = 0.4(0.12) + w_B(0.1) + (0.6 - w_B)(0.06) \sim\sim\sim \{3\}$$

Complete portfolio

$$\text{Slope} = \frac{E(r_0)w_R + (1-w_R)(0.03) - 0.03}{w_R(0.16)} = 0.35 \sim\sim\sim \{1\}$$

$$U = E(r_0)(w_R) + (1 - w_R)(0.03) - \frac{3}{2}(w_R)^2(0.16)^2$$

$$\frac{dU}{dw_R} = E(r_0) - 0.03 - 3w_R(0.16)^2 = 0 \leftarrow \text{maximize utility}$$

$$w_R = \frac{E(r_0) - 0.03}{3(0.16)^2}$$

$$E(r_0) = 0.03 + 3(0.16)^2 w_R \sim\sim\sim \{2\}$$

Substitute {2} into {1}

$$(0.03 + 0.0768w_R)(w_R) - 0.03w_R = 0.35(0.16)w_R$$

$$w_R = 0.7292$$

(portion to invest in optimal risky portfolio)

$$E(r_0) = 0.03 + 3(0.16)^2(0.7292) = 0.086$$

(expected return of optimal risky portfolio)

Substitute $E(r_0) = 0.086$ into {3}

$$0.086 = 0.048 + w_B(0.1) + (0.6 - w_B)(0.06)$$

$$w_B = 0.05$$

$$w_C = 0.6 - 0.05 = 0.55$$

(portion in optimal risky portfolio)

For complete portfolio

$$\text{Portion for risk free asset D} = 1 - 0.7292 = 0.2708$$

$$\text{Portion for asset A} = 0.7292(0.4) = 0.2917$$

$$\text{Portion for asset B} = 0.7292(0.05) = 0.03646$$

$$\text{Portion for asset C} = 0.7292(0.55) = 0.4010$$

$$E(r) = 0.7292(0.086) + 0.2708(0.03) = 0.07083$$

$$\sigma = 0.7292(0.16) = 0.1167$$

The proportion shown above is to minimize risk with highest return in optimal risky portfolio, then distributed among optimal risky and risk free asset D based on investor's risk aversion of 3.

Model Solution 3:

$$\text{Given: } A=3; w_A=0.4; \sigma_p=0.16; \frac{E(r_P)-r_f}{\sigma_P} = 0.35$$

From D, we can conclude $r_f = 0.03$

$$\frac{E(r_P)-r_f}{\sigma_P} = 0.35 \Rightarrow \frac{E(r_P)-0.03}{0.16} = 0.35 \Rightarrow E(r_P) = 0.086$$

$$E(r_P) = w_A E(r_A) + w_B E(r_B) + w_C E(r_C) = (0.4)(0.12) + w_B(0.1) + w_C(0.06)$$

$$\Rightarrow 0.086 = 0.048 + w_B(0.1) + (0.6 - w_B)(0.06)$$

$$\Rightarrow 0.038 - 0.036 = w_B(0.1 - 0.06)$$

$$\Rightarrow w_B = 5\%$$

$$\Rightarrow w_C = 0.6 - w_B = 55\%$$

\Rightarrow optimal investment in risky portfolio, assuming $U = E(r) - \frac{1}{2}A\sigma^2$

$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.086 - 0.03}{(3)(0.16)^2} = 72.9167\%$$

$$\Rightarrow \text{Investment in risk-free asset} = 1 - y^* = 27.0833\%$$

Optimal investment plan:

-Invest in A = $(0.4) \times (0.729167) = 29.167\%$ of funds

-Invest in B = $(0.05)(0.729167) = 3.646\%$ of funds

-Invest in C = $(0.55)(0.729167) = 40.104\%$ of funds

-Invest in D = 27.0833% of funds

Expected return of complete portfolio:

$$E(r_C) = r_f + y^*[E(r_P) - r_f] = 0.03 + 0.729167[0.086 - 0.03] = 7.083\%$$

Std deviation of complete portfolio (Portfolio will have same sharpe ratio as the risky portfolio)

$$\frac{E(r_C) - r_f}{\sigma_C} = 0.35 \Rightarrow \sigma_C = \frac{0.07083 - 0.03}{0.35} = 11.667\%$$

Model Solution 4:

$$(1) S_P = \frac{E(r_P) - r_f}{\sigma_P} = \frac{E(r_P) - 0.03}{0.16} = 0.35 \Rightarrow E(r_P) = 0.086$$

(2) P_B – proportion in B in the optimal risky portfolio

P_C – proportion in C in the optimal risky portfolio

$$(3) P_B \times 0.1 + P_C \times 0.06 + 0.4 \times 0.12 = 0.086 \quad \}$$

$$P_B + P_C = 0.6 \quad \} \rightarrow P_B = 0.05; P_C = 0.55$$

(4) y – Proportion invested in the optimal risky portfolio

$$(5) y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.086 - 0.03}{3 \times 0.16^2} = 72.917\%$$

(6) The proportion invested in A: $72.917\% \times 0.4 = 29.167\%$

The proportion invested in B: $72.917\% \times 0.05 = 3.646\%$

The proportion invested in C: $72.917\% \times 0.55 = 40.104\%$

The proportion invested in D: $100\% - 72.917\% = 27.083\%$

The overall expected return: $0.72917 \times 0.086 + 0.27083 \times 0.03 = 7.083\%$

The SD of the overall portfolio: $0.72917 \times 0.16 = 0.11667$

Model Solution 5:

$$\text{Slope of CAL} = 0.35 = \frac{E(r_P) - r_f}{\sigma_P}$$

$$r_f = 3\% \quad \sigma_P = 16\%$$

$$E(r_P) = w_A E(r_A) + w_B E(r_B) + w_C E(r_C), \quad \sum w = 1$$

$$= 0.4(0.12) + w_B(0.1) + w_C(0.06)$$

$$= 0.4(0.12) + w_B(0.1) + (0.6 - w_B)(0.06)$$

$$\Rightarrow 0.35 = \frac{0.048 + 0.1w_B + 0.036 - 0.06w_B - 0.03}{0.16}$$

$$= \frac{0.054 + 0.04w_B}{0.16}$$

$$w_B = 0.05$$

$$\Rightarrow w_C = 0.55 \Rightarrow E(r_p) = 0.086$$

$$\text{Complete portfolio: } E(r_c) = r_f + y(E(r_p) - r_f), \quad y = \% \text{ in risky asset}$$

$$= 0.03 + 0.056y$$

$$\text{Utility: } U = E(r_c) - \frac{1}{2} A \sigma_c^2$$

$$= 0.03 + 0.056y - \frac{1}{2} (3) \sigma_c^2$$

$$\sigma_c = y \sigma_p = 0.16y$$

$$\Rightarrow U = 0.03 + 0.056y - \frac{3}{2} (0.16^2) y^2$$

$$U' = 0.056 - 3/2 (0.16^2)(2y) = 0$$

$$y^* = 0.7292$$

$$\text{Invest } (0.7292)(0.4) = 29.17\% \text{ in Asset A}$$

$$(0.7292)(0.05) = 3.65\% \text{ in Asset B}$$

$$(0.7292)(0.55) = 40.10\% \text{ in Asset C}$$

$$1 - 0.7292 = 27.08\% \text{ in Asset D}$$

The expected return of the complete portfolio is $E(r_c) = 0.03 + 0.056y = 7.08\%$

The expected risk of the complete portfolio (in term of σ) = $\sigma_c = y \sigma_p = 0.1167$

Examiner's Comments:

This part was generally responded to well, with a many candidates receiving full credit.

The most common mistake was ignoring the fact that the item instructed candidates to give the overall expected risk and return results of the "optimal investment plan", which the item specified included some of each asset (A, B, C, & D). Many candidates omitted these calculations for $E(r_c)$ and σ_c . Some other responses that lost points related to this mistake include using the expected return on the Optimal Risky Portfolio of $E(r_p) = 8.6\%$ as the expected return on the Optimal Complete Portfolio, or stating that there wasn't enough information to calculate the standard deviation of the Optimal Complete Portfolio.

The next most common mistake was in calculating the weights given to assets B and C in the Optimal Risky Portfolio. When setting up the equation to solve for the weight in asset B, candidates had to restate the weight given to asset C in terms of the weight given to asset B as in the following equation:

$$0.086 = 0.40 * 0.12 + w_B * 0.10 + (1 - 0.40 - w_B) * 0.06.$$

However, many candidates set up the equation as follows:

$$0.086 = 0.40 * 0.12 + w_B * 0.10 + (1 - w_B) * 0.06$$

Another common mistake was made in calculating y^ , the percentage of the Optimal Complete Portfolio that is invested in the Optimal Risky Portfolio. For example, some candidates wrote the y^* formula with an extra factor of 2 in the denominator, whether they differentiated to find the formula themselves or simply stated the formula from memory. This is most likely a result of erroneously eliminating the $\frac{1}{2}$ factor during differentiation of $U = E(r_c) - \frac{1}{2} A \sigma_c^2$. Other candidates wrote the correct formula but simply made calculation errors in solving for y^* .*

Other common mistakes included candidates failing to realize they had all the information necessary to fully respond to the item. Many candidates stated they needed to know the standard deviations of the individual assets

and made assumptions, while others erroneously calculated the standard deviations of the individual assets assuming each one lies on the CAL.

Part b:

Model Solution 1:

Optimal risky portfolio with asset A, B and C are mean variance efficient, so it is optimal. Other combination of A, B and C would have lower expected return or higher risk which are not efficient and thus not the choice for rational investors. When combining optimal risky portfolio with risk free asset D, it would depend on each investor's risk aversion. Risk taking investor would invest more in optimal risky portfolio and less in asset D.

Model Solution 2:

Portfolio selection can be separated into two broad categories (i.e. "separation property")

1. Selection of the optimal risky portfolio. This is the same for all investors
2. Allocation of funds between the optimal risky portfolio & the risk-free asset. This will depend on the investor's risk preferences

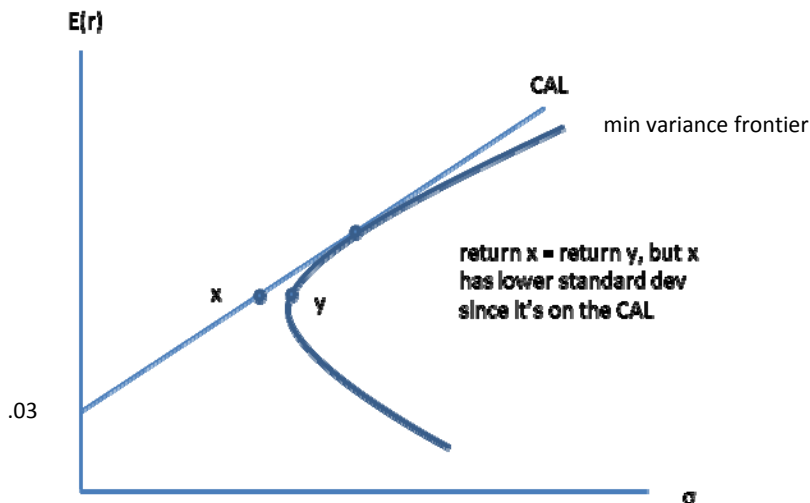
=> Asset D is basically the risk-free asset & therefore the proportion invested in it will vary based on risk preferences. The proportions in the other assets (forming the optimal risky portfolio) will be the same for all investors

Model Solution 3:

- (1) The first step for determining complete portfolio for an investor is the selection of optimal risky portfolio. This will be same for all investors.
- (2) The separation among investor choices for their complete portfolio is solely a function of their personal preference for risk.

Model Solution 4:

The best risk return portfolio is determined by where the CAL is tangent to the minimum variance frontier. This is the max slope available. In order to keep this same slope, we must move along the CAL using risk free assets instead of changing the mix in the risky portfolio.



Model Solution 5:

The weights of A, B & C in risky portfolio represent the combination of these three assets that will maximize the sharpe ratio (slope of the CAL). So all investors (with these 3 assets) should hold a risky portfolio with these weights. What will vary is the amount assigned to the risky versus risk free to form the complete portfolio. This will be based on the individuals risk aversion.

Model Solution 6:

In efficient markets, all investors should hold the same risky portfolio. The only reason for any change in the risky portfolio is due to taxes or some other special risk factor such as age or profile of the person. Since all people should hold the same risky portfolio, the only difference in the overall portfolio is the proportion held between the risky portfolio & the risk free asset. That varies due to risk averse-ness. So the more risk averse a person is, the more they will hold in D.

Model Solution 7:

The portion invested in Asset D will be the only difference between plans for investor with the same assets because of the separation property. The determination of the optimal complete portfolio is comprised of 2 independent steps

1. Determine the optimal risky portfolio. This is a technical exercise and will be the same for each investor.
2. Determine proportion invested in the optimal risky portfolio based on the individuals risk aversion level.

This explains why the amount invested in Asset D will be vary based on the individuals risk aversion. The optimal risky portfolio is the same for everyone.

Examiner's Comments:

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*This part was generally responded to well, with most candidates receiving full credit. Any reasonable answer that fully responded to the item received credit. The most common reason that candidates lost points was due to a response that lacked sufficient detail for full credit. Given the keyword "Explain" we were looking for a more complete explanation of the Separation Property (using the key term Separation Property was not necessary for credit). Many candidates who lost points explained why the Optimal Risky Portfolio would be the same for everyone, or they explained how the amount invested in asset D would vary based on the individual's risk aversion, but they didn't put both concepts together to form a full explanation. The key here was completeness of the explanation, not necessarily length.*  
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Question 2

Solution 1

Implied 1 year spot rate = $\$1000 / \$952.38 \rightarrow 5.00\%$

Implied 2 year spot rate = $\$1000 / \$902.73 \rightarrow 5.25\%$

Implied 3 year spot rate = $\$1000 / \$845.55 \rightarrow 5.75\%$

Implied 4 year spot rate = $\$1000 / \$807.23 \rightarrow 5.50\%$

Therefore:

Forward rate between 1 & 2 = $(1.0525)^2 / 1.05 - 1 = 5.50\%$

Forward rate between 2 & 3 = $(1.0525)^3 / (1.05)^2 - 1 = 6.76\% \rightarrow$ higher than given forward rate of 5.50%

Forward rate between 3 & 4 = $(1.055)^4 / (1.0575)^3 - 1 = 4.75\%$

Buy 3 year bond, sell 2 year bond and enter into a forward rate agreement to receive floating and pay fixed 5.50%.

$$\text{Profit} = \$1000(0.067 - 0.055) = \$12.60$$

Solution 2

Year	Forward Rate	Implied Bond Price
1	5.0%	\$952.38
2	5.5%	\$907.73 = $\$1000 / (1.05 * 1.055)$
3	5.5%	\$855.67 → Arbitrage
4	6.0%	\$807.23

Short a 2 year bond and buy (902.73 / 845.55) of 3 year bonds with the proceeds. This will allow you to earn $(902.73 / 845.55 - 1) = 6.8\%$ between time 2 and 3 on \$1,000 of principal. Short a forward agreement for year three which will cost 5.5%.

$$\text{Profit} = \$1000(6.8\% - 5.5\%) = \$12.62 \text{ undiscounted}$$

Solution 3

Assume you can borrow \$845.55 using forward rates for 3 years. Buy the 3 year bond at 5.75% yield.

At the end of 3 years, get \$1,000 back from maturing bond and pay back the loan:

$$\$1000 - \$845.55 * (1.05 * 1.055 * 1.055) = 11.83$$

Solution 4

Year	Implied Forward Rate
1	5.00%
2	5.50%
3	6.76%
4	4.75%

Since the f_3 implied by the bond prices is higher than the actual forward rate, an arbitrage opportunity exists. I assume I can borrow at the forward rates shown at time 0 so I borrow $\$1000 / (1.05 * 1.055 * 1.055) = \855.67 . Use this money to buy $855.67 / 845.55 = 1.012$ 3-year bonds.

At time 3, bonds are worth \$1,012. Pay back loan of \$1,000 and difference is \$12 profit.

Solution 5

Strategy:

- 1) Sell $845.55 / 807.23 = 1.0475$ units of 4-year bond
- 2) Buy the 3-year bond and use an FRA to reinvest at the 6% forward rate

	Now	3-years	4-years
3-year bond	\$845.55	\$1,000	$\$1000 * 1.06 = \1060
1.0475 units of 4-year bond	-\$845.55		$-\$1000 * 1.0475 = -\$1,047.50$
Net Investment	0		\$12.53 profit

Solution 6

$$Y_3 = (1000/845.55)^{(1/3)} - 1 = 5.75\% \neq [1.05 * 1.055 * 1.055]^{(1/3)}$$

Short a 3-year bond base on the given forward rates with price = $\$1000 / (1.0533)^3 = \855.67

Buy the underpriced 3-year bond at \$845.55

Gain a risk-free profit of $(5.75\% - 5.33\%) = 0.42\%$

Solution 7

Assume the forward rate for year 3 at time 0 will actually be the 1 year spot rate in 2 years.

Sell \$845.55 worth of the 2 year bond and buy the 3 year bond for \$845.55. Net investment at time 0 is \$0. At time 2 the 2 year bond will mature and you will owe $\$936.66 = 1000/902.73 * 845.55$

At this point you should sell the original 3 year bond (1 year left now). The price you will receive based on the forward rate is $\$947.87 = 1000/1.055$

Arbitrage profit = $\$947.87 - \$936.66 = \$11.21$

Solution 8

Price at forward rates:

1. $\$952.38 = 1000/1.05$
2. $\$902.73 = 1000/(1.05*1.055)$
3. $\$855.67 = 1000/(1.05*1.055*1.055) \rightarrow$ Mispriced
4. $\$807.23 = 1000/(1.05*1.055*1.055*1.06)$

Assume you can borrow at the current rates and borrow the money and buy the 3 year bond. The PV of profit is equal to the discounted price of the bond less the cost.

PV Profit = $\$855.67 - \$845.55 = \$10.12$

Solution 9

<u>Year</u>	<u>Implied Bond Price</u>
1	\$952.38
2	$\$907.73 = \$1000/(1.05*1.055)$
3	$\$855.67 \neq \845.55
4	\$807.23

We note the 3 year bond is mispriced, presenting an arbitrage opportunity.

Sell $845.55/902.73$ 2 year bonds at time 0 = \$845.55, agree to payback at time 3 and lock in the 5.5% rate from time 2 to 3

Buy the 3 year bond at time 0 = $-\$845.55$

Time 2 = $-\$936.66$

Time 3 = $-\$936.66*1.055 = -\988.18

Profit = $\$1,000 - \$988.18 = \$11.82$

Solution 10

<u>Year</u>	<u>Yield</u>	<u>Yield implied by Forward Rates</u>
1	5.00%	5.00%
2	5.25%	5.25%
3	5.75%	5.33%
4	5.50%	5.50%

The strategy would be to buy bonds with maturity 3 years and sell short the bond with maturity 4 years

Profit = $\$807.23$ (from selling short bond with $t=4$) $* 1.0575^3 * 1.06 - \$1000$ (payment of bond sold short at $t=4$) = \$11.92

Solution 11

3 year bond – [$\$1000/\845.55] = 1.1827

$(1.05 \times 1.055 \times 1.055) = 1.1687 \neq 1.1827 \rightarrow$ arbitrage

Assuming we can take a short position in the forward rates of 1.05, 1.055 and 1.055 for 3 years. Take the proceeds to purchase a 3 year bond resulting in $1.1827 - 1.1687 = 0.014$ profit

Solution 12

Sell the 2 year bond and buy the 3 year bond to sell at 2 years

Assume investing \$1,000 at t=0

$\$1000/\$902.73 = 1.1078$ of 2 year bond

$\$1000/\$845.55 = 1.1827$ of 3 year bond

Pay $1000(1.1078) = \$1107.75$ at t=2

Sell the 3 year bond at t=2 for $\$1000(1.1827) / (1.055) = \1121.04 (assume we realize the 3rd year forward rate given in the problem)

Profit = $\$1121.04 - \$1107.75 = \$13.29$

Solution 13

The 3 year bond is underpriced. Arbitrage strategy is to buy 3 year bond, short 1.0475 of the 4 year bond and enter into FRA for \$1000 and rate of 6% from time 3 to time 4

Time 0 = buy 3 year, short 4 year, enter into FRA for \$0 cost

Time 3 = +\$1000 – \$1000 (3 year bond matures, FRA starts)

Time 4 = +\$1060 – \$1.047.5 (4 year bond and FRA mature)

Profit = \$12.5

Examiner's Comments:

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The question asked for two things:

1. Design an arbitrage strategy, and
2. Calculate a profit

**Designing an Arbitrage Strategy**

In order to design an arbitrage strategy, first an arbitrage opportunity must be identified. There are a couple of ways that this could be observed. The clearest was to calculate the expected price of each bond using the given forward rates and compare the expected price with the given price.

As an example,

For the 1-year bond, the expected price is:

$\$1,000/1.05 = 952.38$

For the 3-year bond, the expected price is:

$\$1,000/(1.05 \times 1.055 \times 1.055) = 855.67$

Continuing this for all four bonds, we get:

| Maturity (years) | Given Price | Expected Price |
|------------------|-------------|----------------|
| 1                | \$952.38    | \$952.38       |
| 2                | \$902.73    | \$902.73       |
| 3                | \$845.55    | \$855.67       |
| 4                | \$807.23    | \$807.23       |

Only the 3-year bond is mispriced.

An alternative way to identify the mispricing was to calculate the implied forward rates from the given bond prices. As follows:

$$f_1 = \$1,000/\$952.38 - 1 = 5.0\% \text{ (equivalent to 5.0\% stated in the problem)}$$

$$f_2 = \$952.38/\$902.73 - 1 = 5.5\% \text{ (equivalent to 5.5\% stated in the problem)}$$

$$f_3 = \$902.73/\$845.55 - 1 = 6.8\% \text{ (different from 5.5\% stated in the problem)}$$

$$f_4 = \$845.55/\$807.23 - 1 = 4.7\% \text{ (different from 6.0\% stated in the problem)}$$

Many candidates calculated these and then stated that the three-year bond was overpriced, which was correct. Some candidates calculated these and then stated that the three-year bond was overpriced and that the four-year bond was underpriced. This is incorrect. The four year bond was priced correctly; the low  $f_4$  was simply the offset to the high  $f_3$  with no resulting mispricing for the four-year bond. The graders chose not to take off for stating that the four year bond was underpriced as long as it was also stated that the three year bond was overpriced. Students should be aware of this common mistake; future exam graders may choose to deduct for this.

Note that if the arbitrage strategy developed would not work because it completely relied on the four-year bond being "mispriced", then points were deducted for not having a workable strategy. This affected very few students since the vast majority of students who stated that both the three- and four-year bonds were mispriced included purchasing the three-year bond as part of their strategy.

When grading whether the students had designed a workable arbitrage strategy, the graders looked for several key elements:

1. That the underpriced bond (3-year bond) was purchased.
2. That interest rate and liquidity risks were completely eliminated.
3. That amounts of investment were determined

#### **Further discussion about Risk Elimination**

Some candidates simply purchased the 3-year bond and held it to maturity. This strategy provides a profit, but it is not an arbitrage profit because you are fully exposed to liquidity/interest rate risk during the three-year period. If you needed cash and had to sell the bond early, you would be subject to the prevailing interest rates at the time and may not get the profit you expect.

Some candidates purchased the 3-year bond and then entered an FRA to lock in paying the 5.5% interest rate in year three on a principal of \$947.86. This strategy provides an arbitrage profit at the beginning of year 3 if you make it to that point, but the deal is exposed to interest/liquidity risk for the first two years.

Some candidates purchased a 3-year bond and shorted a 2-year bond and then sold the 3-year bond at the beginning of year 3, so their timeframes match up, but when they sell that bond at the beginning of year 3, they are fully at the mercy of the current interest rates unless they have locked in an agreement to use the current given forward rates at that time. The graders saw lots of different flavors of this. Another common related strategy was to buy a three-year bond and fund it by selling a one-year bond and then selling another 1-year bond at the end of the first year and then selling another 1-year bond at the end of the second year. For any strategy that involved buying or selling any assets in the future, the strategy needed to include that the current forward rates were being locked in for those investments.

Some candidates simply borrowed the money to purchase the three year bonds. Credit for this strategy relied upon ensuring the money was being borrowed for three years and that the rates matched the ones given in the problem.



### Further discussion about Amounts

It was not enough for a candidate to simply say, “Borrow money and invest in 3-year bonds.” Either amounts of money borrowed and amounts invested in bonds needed to be given or else cashflows needed to be shown so that it was clear that interest rate risk was being eliminated. Quite a few candidates showed amounts that eliminated a lot of risk but did not make the deal risk-free. For example, they purchased a \$1,000 3-year bond and shorted a \$1,000 4-year bond and entered an FRA to borrow \$1,000 at the beginning of year 4 using the current forward rates. One hole in this strategy is that there is an upfront investment of  $\$845.55 - \$807.23 = \$38.32$ . That there is an upfront investment is an indicator that liquidity risk has not been eliminated. If the investor needs to get his hands on this capital before the contract is over, he will have to reverse the contract at current market rates.

(Not needed)

Though it was not required, many candidates included a table of their cash flows and this greatly helped the graders see whether the candidate understood what was going on. In fact, there were many times where candidates didn’t describe what their strategy was well with words, but the graders gave them some credit because they showed their cash flows and these made sense.

### Calculating a Profit

The exam did not specify whether the calculated profit needed to be in dollars or in percent, so the graders accepted either. For those candidates providing a percent profit, most did not define what the percent was of or they were unclear (like, they said “percent of investment” but they had just constructed a strategy with \$0 net investment!) the graders did not take off for not explaining what the percent profits were a percent of. It was generally clear from the calculations.

Generally, any profit that followed the designed strategy (even if the strategy was flawed) and was free of mathematical errors received full credit for that part of the problem.

Many candidates did not give any type of resultant profit at all and simply stopped after the strategy design.

### A Few Other Notes

The question specified that interest rates were compounded annually, so candidates using any other type of compounding lost a bit of credit. This didn’t happen often. The most common type of compounding to erroneously be employed was continuous compounding.

A handful of candidates tried to use the formula for valuing a Forward Rate Agreement. This formula finds a value for a forward rate agreement when the forward rate being agreed to in a contract is different from the expected forward rate. Most candidates were entering FRAs using the given forward rates, but were then trying to use this formula to extract a non-zero value, which didn’t work. However, one candidate did show us that it could be used appropriately within the context of the problem.

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### Question 3:

Part a:

Model Solution 1:

weak form – this states that stock prices reflect all historical market data, such as trading volume and historical prices.

If the new CFO believes in the weak form, then he may think that current investment manager was providing benefit in his analysis by gaining insight through non-market data  
semi-strong form – says that stock prices reflect all publicly available information, market data plus everything else the public has access to, such as news reports and earnings reports. If the new CFO believes in this form, then the only way the investment manager is providing benefit would be if he had insider information. Since that's illegal then he would likely make the manager stop his fundamental analysis  
strong form – says stock prices reflect all information, including insider information. In this case the CFO would conclude that fundamental analysis could provide no benefit. The investment manager would be told to stop.

#### Model Solution 2:

weak-form – stock prices reflect everything that can be gained from analysis of past stock prices  
- not relevant b/c fundamental analysis uses more than just stock prices  
semi-strong form – stock prices reflect all info that can be gained from publicly available information  
- would violate semi-strong form since fundamental analysis uses publicly available info  
strong form – stock prices reflect all info about a firm, both public and private information  
- violates strong form since violates semi-strong form

#### Model Solution 3:

1. Weak form – prices already reflect all information available on past market data. This tells the CFO that you can't earn excess returns by analyzing past stock prices (technical analysis). Not relevant here since fund manager uses fundamental analysis.
2. Semi-strong form – prices already reflect all publicly available information about assets/companies. This tells the CFO that fundamental analysis won't earn excess returns unless you have non-public information about a company.
3. Strong form – all information is already reflected, even non-public information. It is unlikely that strong-form efficiency holds, so fundamental analysis could work if you have non-public information.

#### Model Solution 4:

Weak – Prices reflect all info that can be obtained by examining historical market price and return data  
- relevance is that inv. man. can't select stocks based on past returns to get abnormal returns  
Semi-strong – Prices reflect all public info about a company's prospects  
- rel: Inv. man. can't get abnormal returns through fundamental analysis of the firm's prospects unless you have unique insight  
Strong – Prices reflect all relevant info about the firm, whether publicly known or not  
Rel: if this is true, should not try to select stocks to have abnormal returns. Passive inv. strategy will work just as well & be cheaper.

#### Model Solution 5:

Weak- stock prices reflect all information that can be derived from trading data

- fundamental analysis uses fundamentals of a firm to attempt to create a return above market, investment manager believes weak form EMH holds, this agrees with CFO's opinion

semistrong – stock prices reflect all information that is publicly available

- since fundamental analysis uses publicly available information, Inv. mgr does not believe EMH semistrong holds, CFO disagrees. CFO may think active portfolio management costs too much for company.

strong – stock prices reflect all information, even that which is not publicly available

- CFO & investment manager likely disagree about market efficiency in the strong form since inv. mgr trying to profit from semistrong form.

#### Model Solution 6:

Weak form: market incorporates all previous information contained in past stock movement.

CFO: "Well, at least you are not using technical analysis of dips and valleys and other techniques that mean nothing for the future."

Semi-strong form: market price includes all publicly available information about the firms' prospects.

CFO: "here's your problem. You don't have access to any information that a thousand other analysis haven't already pored over. Everything you are studying is factual and relevant, but the market has already responded to every piece of it."

Strong form: market prices reflect all information, including insider information

CFO: "I'm such a believer in market efficiency, that even insider info, like the CEO stepping down or the not-yet-released reserve numbers are there. And you don't even have access to that. Passive strategy is all we can do."

#### Model Solution 7:

Weak form – this states that stock prices reflect all past trading history ex. past stock prices

This isn't very relevant to fundamental analysis, as its typically used more in technical analysis

semi-strong form – states that all publically available information is reflected in the stock price

This is relevant because fundamental analysis uses all available information.

strong form – states all public and private (insider info) is reflected in the stock price.

This isn't very relevant because it's been proven that insider information can be used to achieve greater returns.

Part b:

The following reasons were given credit:

- Because of the wide variance in both returns compared to the relatively small difference in returns, it's hard to say whether the difference is due to randomness or not.
- Active portfolio management has costs associated with it. → rates of return may be lower after adjusting for expenses (likely the case in our situation since the returns are only slightly higher than the S&P 500's)
- Lucky Event Issue: Perhaps the investment manager has just gotten lucky. If they have a 50/50 chance of beating the market every year, based on binomial distribution, they have a 12.5% chance of beating it every year over 3 years
- S&P 500 is only a proxy for the market and only includes ~77% of securities so the actual market returns may vary from the index's returns.
- We were not given the βs of the current portfolio. The company may have been taking on more risk to achieve higher returns.
- "Excess Returns" may be a liquidity premium for holding less liquid assets.

Examiner's Comments:

Part a:

Most candidates correctly described the three forms of the efficient market hypothesis. There were a surprising variety of interpretations of what it means to "assess the relevance... to the CFO's evaluation of the current investment manager's strategy." We gave credit for any reasonable interpretation as long as the answer demonstrated understanding of the relevant learning objective (A.9) and of the associated knowledge statements.

Several candidates were able to list the three forms of the EMH but lost significant credit because they completely misunderstood the implications of the EMH for portfolio management. Many of these candidates thought that the three forms of the EMH were investment strategies, rather than theories about what information is reflected in current asset prices. For example, these candidates tended to say things like "the weak form of the EMH says that stock prices are based on past trading data. If the current investment manager believed in the weak form of the EMH then he would be using technical analysis instead of fundamental analysis."

Another common error was to describe the weak form of the EMH as saying that the current market price reflected all historical data. In order to get full credit, a candidate needed to narrow this down to market trading.

Part b:

Candidates were typically successful at presenting at least 2 reasons that the investment manager's position may not be valid. Many candidates who failed to earn full credit on this question appeared to be trying to regurgitate a list they had memorized from the readings instead of responding to the specific argument described in the question. The specific scenario described in the question was not drawn directly from the readings, and the fully successful candidates were the ones who analyzed the argument presented and synthesized appropriate information from the readings to rebut it.

Candidates frequently tried to include an argument labeled “selection bias.” This term was applied to a surprising variety of arguments, none of which were valid in this case. Although the readings identify selection bias as a reason why it is difficult to test market efficiency in general, it is not a valid rebuttal of the specific argument presented in the problem – the argument includes investment returns for the single investment manager under consideration and for all three years that the company has been using fundamental analysis.

In many cases, candidates listed arguments that actually supported the investment manager’s decision, rather than rebutting it. For example, several candidates listed market anomalies that are often cited as evidence that markets are not efficient. Others listed evidence of investor irrationality based on the behavioral critique of the EMH.

**Question 4:**

Part a:

( a ) Calculate McCauley Duration:

| <u>t</u> | <u>% Loss</u> | <u>Total Loss</u><br>\$5,000,000<br><u>Proj. Loss</u> | <u>Discount Rate</u><br>1.04<br><u>PV(Loss)</u> | <u>t*PV(L)</u>      |
|----------|---------------|-------------------------------------------------------|-------------------------------------------------|---------------------|
| <u>1</u> | <u>0.30</u>   | <u>1,500,000</u>                                      | <u>1,442,308</u>                                | <u>1,442,307.69</u> |
| <u>2</u> | <u>0.35</u>   | <u>1,750,000</u>                                      | <u>1,617,973</u>                                | <u>3,235,946.75</u> |
| <u>3</u> | <u>0.25</u>   | <u>1,250,000</u>                                      | <u>1,111,245</u>                                | <u>3,333,736.35</u> |
| <u>4</u> | <u>0.10</u>   | <u>500,000</u>                                        | <u>427,402</u>                                  | <u>1,709,608.38</u> |
|          | <u>1.00</u>   | <u>5,000,000</u>                                      | <u>4,598,929</u>                                | <u>9,721,599</u>    |
|          |               |                                                       | <u>Macaulay</u><br><u>Duration =</u>            | <u>2.11388</u>      |

$( 9,721,599 / 4,598,929 ) = 2.11388$

Alternate 1 – calculated with only showing % of Loss:

| <u>t</u> | <u>% Loss</u> | <u>% Loss</u> | <u>PV(Loss%)</u> | <u>t*PV(L%)</u> |
|----------|---------------|---------------|------------------|-----------------|
| <u>1</u> | <u>0.300</u>  | <u>0.300</u>  | <u>0.2885</u>    | <u>0.2885</u>   |
| <u>2</u> | <u>0.350</u>  | <u>0.350</u>  | <u>0.3236</u>    | <u>0.6472</u>   |
| <u>3</u> | <u>0.250</u>  | <u>0.250</u>  | <u>0.2222</u>    |                 |

|          |              |              |                   |                |
|----------|--------------|--------------|-------------------|----------------|
|          |              |              |                   | <u>0.6667</u>  |
| <u>4</u> | <u>0.100</u> | <u>0.100</u> | <u>0.0855</u>     | <u>0.3419</u>  |
|          | <u>1.000</u> | <u>1.000</u> | <u>0.9198</u>     | <u>1.9443</u>  |
|          |              |              | <u>Macaulay</u>   |                |
|          |              |              | <u>Duration =</u> | <u>2.11388</u> |

$( 1.9443 / 0.9198 ) = 2.11388$

Alternate 2 – Calculated assuming Loss Evaluation Date is 12/31/13. Losses already paid in 2013 – calculate the Macaulay Duration of the loss reserves for accident year 2013. (i.e. would make sense that the evaluation date is at the end of 2013 and consistent with Feldblum paper.

|          |             |                  |                   |                  |
|----------|-------------|------------------|-------------------|------------------|
| <u>1</u> | <u>0.35</u> | <u>1,750,000</u> | <u>1,682,692</u>  | <u>1,682,692</u> |
| <u>2</u> | <u>0.25</u> | <u>1,250,000</u> | <u>1,155,695</u>  | <u>2,311,391</u> |
| <u>3</u> | <u>0.1</u>  | <u>500,000</u>   | <u>444,498</u>    | <u>1,333,495</u> |
|          | <u>0.7</u>  | <u>3,500,000</u> | <u>3,282,886</u>  | <u>5,327,577</u> |
|          |             |                  | <u>Macaulay</u>   |                  |
|          |             |                  | <u>Duration =</u> | <u>1.6228</u>    |

$( 5,327,577 / 3,282,886 ) = 1.6228$

Alternate 3: Same Answer using loss percentages: (Evaluation Date 12/31/2013)

| <u>t</u> | <u>% Loss</u> | <u>% Loss</u> | <u>PV(Loss%)</u>  | <u>t*PV(L%)</u> |
|----------|---------------|---------------|-------------------|-----------------|
| <u>1</u> | <u>35%</u>    | <u>35%</u>    | <u>0.3365</u>     | <u>0.3365</u>   |
| <u>2</u> | <u>25%</u>    | <u>25%</u>    | <u>0.2311</u>     | <u>0.4623</u>   |
| <u>3</u> | <u>10%</u>    | <u>10%</u>    | <u>0.0889</u>     | <u>0.2667</u>   |
|          | <u>70%</u>    | <u>70%</u>    | <u>0.6566</u>     | <u>1.0655</u>   |
|          |               |               | <u>Macaulay</u>   |                 |
|          |               |               | <u>Duration =</u> | <u>1.6228</u>   |

$( 1.0655 / 0.6566 ) = 1.6228$

Examiner's comments:

The following were common mistakes made for this part of the question:

- (1) Used 1.02 to discount => Answer 2.13 Mistakenly used the Risk Free Rate to calculate the Duration where the return on the company's assets (4%) should be used.
- (2) Assumed losses paid at Mid-year => Answer 1.614. Problem clearly says to assume losses paid at the end of each development year.
- (3) Calculating  $t \cdot PV(L\%)$  and not dividing by sum of  $PV(Loss\%)$

Part b:

Exam 9, question 4 part B (1 point)

"Illustrate to the management team how to immunize the loss reserve liability of accident year 2013 from interest rate fluctuations and briefly describe two limitations associated with the proposal."

- Model solutions
  - Duration matching -
    - Invest in bonds/assets portfolio with ultimate value \$5 Million with an average duration of 2.113. *(if assuming reserves are at beginning of year 2013)*
    - Invest in bonds/assets portfolio with ultimate value \$3.5 Million with an average duration of 1.623. *(if assuming reserves are at end of year 2013)*
    - Invest in bonds/assets portfolio with present value of \$5Mx.92 *(calculated as denominator of part A)* = 4.6M with an average duration of 2.113. *(credit also given if recalculated using expected payments)*
    - Invest in bonds/assets portfolio with present value of \$3.5Mx.938 *(calculated as denominator of part A)* = 3.28M with an average duration of 1.623. *(credit also given if recalculated using expected payments)*
  - Cash flow matching -
    - Invest in four zero-coupon bonds that match payment with the loss payout, \$1.5 Million maturing in the first year, \$1.75 Million maturing in the second year, \$1.25 Million maturing in the third year, and \$0.5 Million maturing in the fourth year. *(assumes reserves are at beginning of year 2013, also could use chart form or present value of cash flows)*
    - Invest in 3 zero-coupon bonds that match payment with the loss payout \$1.75 Million maturing in the first year, \$1.25 Million maturing in the second year, and \$0.5 Million maturing in the third year. *(assumes reserves calculated at end of year 2013, also could use chart form or present value of cash flows)*
  - Duration matching – disadvantage: only protect against small interest rate fluctuations; only protect against parallel shift in term structure; need to rebalance portfolio, does not hold if inflation changes, may have to give up higher yield investments,

- Cash flow matching – disadvantage: high cost to maintain; constraint in bond selection, only protects against nominal values (inflation not covered), does not hold if payments do not come in as expected, may have to give up higher yield investments, may not be possible to find assets with exact cash flows

*Examiner's Comments:*

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Most common mistake was simply describing the technique used to immunize the liability and not taking this question to the higher taxonomy level indicated by "illustrate". For example, just saying "duration matching" or just saying "cash flow matching" or just including a brief description of what cash flow matching is. The intended answer needed form of example including numbers, whether in chart form or listing out the future values or present values of the amounts in either cash flow matching or duration matching.

Another common mistake was to calculate present values for duration matching incorrectly. Not using the wrong interest rate but wrong method such as just discounting the 5M ultimate from time 4 rather than each expected payment from the expected time frame. The simple way to do it is to use the total present value factor calculated for the denominator in part A.

Another extremely common mistake was describing "cash flow matching" limitations when the method of immunization used was "duration matching" or vice versa. Some limitations do apply to both while some do not. For example, illustrating cash flow matching as immunization method and then using a duration limitation such as cumbersome due to need to rebalance.

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**Question 5:**

Model Solution 1:

$$F = \left( P - E - \frac{L}{1 + y} \right) * \frac{cr/(1 + y)}{1 - cr/(1 + y)}$$

$$= \left( 3115 - 1050 - \frac{2100}{1.07} \right) * \frac{.85/(1.07)}{1 - .85/(1.07)}$$

$$= 395.57$$

$$D_F = \frac{a - b + 1}{(1 + y)(a + by + y)} + \frac{1}{1 + y + cr}$$

Current:  $k = a + by$ ,  $a = 0.15$ ,  $b = 0$ ,  $y = 0.07$

$$= \frac{0.15 - 0 + 1}{(1.07)(0.15 + 0 - 0.07)} + \frac{1}{1.07 - 0.85}$$

$$= 17.98$$

$$C = S + P - E - \left( \frac{L}{1 + y} \right)$$



$$= 1400 + 3115 - 1050 - \left(\frac{2100}{1.07}\right)$$

$$= 1502.38$$

$$D_{Total} = \frac{1502.38 + (395.57 * 17.98)}{1502.38 + 395.57}$$

$$= 4.54$$

Proposed:  $k = a + by$ ,  $a = 0.08$ ,  $b = 1$ ,  $y = 0.07$

$$D_F = \frac{0.08 - 1 + 1}{(1.07)(0.08 + 0.07 - 0.07)} + \frac{1}{1.07 - 0.85}$$

$$= 5.48$$

$$D_{Total} = \frac{1502.38 + (395.57 * 5.48)}{1502.38 + 395.57}$$

$$= 1.93$$

Reduction in  $D_{Total} = 4.54 - 1.93 = 2.61$

By changing the pricing strategy so that the target rate of return on surplus is a function of the interest rate, we can severely reduce the duration of the franchise value and thus the total economic value. In this example, we reduced the duration of the total economic value by 2.61 years, from 4.54 to 1.93.

### Model Solution 2:

$P = 3115$        $L = 2100$        $F = 1050$        $S = 1400$        $k = 15\%$   $y = 7\%$     $cr = 85\%$

$$C = S + P - E - \left(\frac{L}{1 + y}\right)$$

$$= 1400 + 3115 - 1050 - \left(\frac{2100}{1.07}\right)$$

$$= 1502.38$$

$$F = \left(P - E - \frac{L}{1 + y}\right) * \frac{cr/(1 + y)}{1 - cr/(1 + y)}$$

$$= \left(3115 - 1050 - \frac{2100}{1.07}\right) * \frac{.85/(1.07)}{1 - .85/(1.07)}$$

$$= 395.57$$

$T = C + F = 1502.38 + 395.57 = 1897.95$

$$D_F = \frac{1-b}{k-y} + \frac{1}{1+y} + \frac{1}{1+y-cr}$$

$$= \frac{1}{0.15-0.07} + \frac{1}{1.07} + \frac{1}{1.07-0.85}$$

$$= 17.98$$

$$D_T = \frac{C * D_C + F * D_F}{T}$$

$$= \frac{(1502.38 * 1) + (395.57 * 17.98)}{1897.95}$$

$$= 4.539$$

To reduce the duration of total economic value, the insurer should make the rate of return, k, vary linearly with the current interest rate, y, such that  $k = a + by$ .

The resulting franchise duration would be:

$$D_F = \frac{1-b}{.15-.07} + \frac{1}{1.07} + \frac{1}{1.07-.85} = \frac{-b}{.08} + \frac{1}{.08} + \frac{1}{1.07} + \frac{1}{.22} = 17.98 - \frac{b}{.08}$$

This results in total duration of:

$$D_{Total} = \frac{(1502.38 * 1) + 395.57 * (17.98 - b/.08)}{1897.95}$$

$$= 4.539 - 2.6052b$$

Therefore, pursuing this strategy will reduce total economic value duration by  $2.6052 * b$  (where b is the coefficient of y in  $k = a + by$ )

*Examiner's comments:*

Common mistakes:

- Many candidates confused y in many of the formulas (which Panning uses to represent the risk-free rate) with k (which Panning uses to represent the target return on surplus). As such, they discounted losses at 15% instead of 7%.
- Many candidates failed to mention that the rate of return on surplus varies with interest rates in Panning's strategy. An explanation of the strategy should consist of more than "change a and b in the formulas".
- A number of candidates presented updated values for a & b that did not adhere to Panning's formula of  $a + by = k$ , or had  $b < 0$ .

Noteworthy:

The inputs in this question were slightly inconsistent, and as such allowed for multiple correct answers. The premium given in the problem should have been slightly higher (about \$3,117 instead of \$3.115) given the other inputs. As such, depending on which formula for Franchise Value the candidate used, they could have one of two answers. We accepted both. There were also candidates that backed into other inputs based on the data given that differed from the inputs in the problem (for instance, many candidates derived a risk-free rate of 7.07% instead of the 7% that was given, which we viewed as correct as well). Any deviations from the model answers that came about due to inconsistencies between inputs were considered correct and awarded credit.

**Question 6:**

The three accepted solutions are below:

| Time Period | Principal  | Coupon | Payment from A to B | LIBOR | LIBOR + .0025 | Payment from B to A | Net CF to A |
|-------------|------------|--------|---------------------|-------|---------------|---------------------|-------------|
| 0.5         | 10,000,000 | 3.0%   | 300,000             | 2.50% | 2.75%         | 275,000             | (25,000)    |
| 1.0         | 10,000,001 | 3.0%   | 300,000             | 2.65% | 2.90%         | 290,000             | (10,000)    |
| 1.5         | 10,000,002 | 3.0%   | 300,000             | 2.80% | 3.05%         | 305,000             | 5,000       |
| 2.0         | 10,000,003 | 3.0%   | 300,000             | 2.95% | 3.20%         | 320,000             | 20,000      |
| 2.5         | 10,000,004 | 3.0%   | 300,000             | 3.10% | 3.35%         | 335,000             | 35,000      |
| 3.0         | 10,000,005 | 3.0%   | 300,000             | 3.25% | 3.50%         | 350,000             | 50,000      |

| Time Period | Principal  | Coupon | Payment from A to B | LIBOR / 2 | LIBOR / 2 + .0025 | Payment from B to A | Net CF to A |
|-------------|------------|--------|---------------------|-----------|-------------------|---------------------|-------------|
| 0.5         | 10,000,000 | 3.0%   | 300,000             | 1.25%     | 1.50%             | 150,000             | (150,000)   |
| 1.0         | 10,000,001 | 3.0%   | 300,000             | 1.33%     | 1.58%             | 157,500             | (142,500)   |
| 1.5         | 10,000,002 | 3.0%   | 300,000             | 1.40%     | 1.65%             | 165,000             | (135,000)   |
| 2.0         | 10,000,003 | 3.0%   | 300,000             | 1.48%     | 1.73%             | 172,500             | (127,500)   |
| 2.5         | 10,000,004 | 3.0%   | 300,000             | 1.55%     | 1.80%             | 180,000             | (120,000)   |
| 3.0         | 10,000,005 | 3.0%   | 300,000             | 1.63%     | 1.88%             | 187,500             | (112,500)   |

| Time Period | Principal  | Coupon | Payment from A to B | LIBOR | LIBOR + .0025 | (LIBOR + .0025)/2 | Payment from B to A | Net CF to A |
|-------------|------------|--------|---------------------|-------|---------------|-------------------|---------------------|-------------|
| 0.5         | 10,000,000 | 3.0%   | 300,000             | 2.50% | 2.75%         | 1.38%             | 137,500             | (162,500)   |
| 1.0         | 10,000,001 | 3.0%   | 300,000             | 2.65% | 2.90%         | 1.45%             | 145,000             | (155,000)   |
| 1.5         | 10,000,002 | 3.0%   | 300,000             | 2.80% | 3.05%         | 1.53%             | 152,500             | (147,500)   |
| 2.0         | 10,000,003 | 3.0%   | 300,000             | 2.95% | 3.20%         | 1.60%             | 160,000             | (140,000)   |
| 2.5         | 10,000,004 | 3.0%   | 300,000             | 3.10% | 3.35%         | 1.68%             | 167,500             | (132,500)   |
| 3.0         | 10,000,005 | 3.0%   | 300,000             | 3.25% | 3.50%         | 1.75%             | 175,000             | (125,000)   |

Examiner's Report:

- The timing of the LIBOR rate was the single most common mistake seen. Candidates would increase the LIBOR by .0015 for use in time period 0.5; however, the article states "For each period, the rate of interest

is set at 0.5% per annum above the 6-month LIBOR rate at the beginning of the period. Interest is paid at the end of the period (p. 148)”

- The exchange of the \$10M in principle at time period 3.0 was included in the cash flows; however, the article states “Note that the \$100M principle is used only for the calculation of interest payments. The principle itself is not exchanged. For this reason it is termed the notional principle, or just the notional (p.149)”
- The use of discounting; however, the question asked just for the cash flows, not the present value of the swap.

The majority of the candidates used the 2.5% 6-month LIBOR without multiplying it by 0.5 to calculate the cash flow. We did accept this response since the purpose of the question was not to test knowledge of the characteristics of the LIBOR.

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Question 7:

Model Solution # 1

- a) The balance of the non-cash assets is in real estate and long-term sovereign debt obligations. These assets are illiquid. To mitigate this source of liquidity risk, the CFO should convert some of the illiquid assets into more liquid ones, thereby managing & balancing the liquidity needs & required returns of the group so as to avoid an illiquidity spiral &/or underperforming investments. The CFO should also consider using discounted/hair-cut valuations of assets when estimating the available liquidity.
- b) The insurance group has all 3 of its reinsurance treaties from the same reinsurer. This is a source of liquidity risk because if the reinsurer defaults, the company will be responsible for funding reinsurer’s portion of the liability. The CFO should diversify its reinsurance portfolio across multiple reinsurance carriers, purchase CAT bonds for its US catastrophe exposure, and do a better job balancing its insurance risk within each nation, both in the geographic sense and the product sense.
- c) Most of the Canadian premiums are in life insurance. There is disintermediation risk in the life insurance market, where policy holders may suddenly cancel their policies & demand the return of their premiums in search of higher returns when interest rates rise. The CFO should mitigate this risk by changing its policy language to include a surrender waiting period and also purchase interest rate hedges.
- d) Quarterly, the CRO uses the most recent Balance Sheet and assumptions of normal economic conditions to set liquidity risk thresholds. Unfortunately, liquidity risk thresholds set under these assumptions & frequency may not be able to withstand periods of liquidity stress. The CFO should use more dynamic, projected cash flows and do so more frequently to set the liquidity risk thresholds. They should examine/consider extreme scenarios and then stress test their liquidity mitigation options under those conditions. The CFO should examine both mid- and long-term liquidity needs, not just daily needs.

Model Solution 2:

Currency risk:

It is troubling that distribution of local currency held does not match distribution of WP, which would be an indicator of liabilities. Even adjusting for difference in type of lines in the different countries, it still seems like too little US & Mexican local currency is being held.

During stress scenarios, FX markets may dry up & regulators may prevent or inhibit transferability of liquidity, which would put severe strain on American & Mexican lines.

The company should redistribute some of its Canadian currency holdings to US & Mexican currencies, reducing currency risk, maintaining independent liquid pool of assets in each country of operations.

Investment risk:

The 43% balance of assets held in real estate & long-term sovereign debt obligations may be seen as highly illiquid, as recent economic events have shown. Turmoil in real estate & sovereign debt markets may lead to these assets only being able to be sold at great loss.

However both Canadian & American lines may lead to sudden huge cash demands (i.e. if a cat happens or if Canadian life policyholders demand refunds). This might then completely deplete currency holdings requiring sale of these illiquid assets at a loss.

To reduce this risk, company should diversify assets (e.g. into stocks, corporate bonds, sovereign debt holdings into more countries) and also diversify insurance operations (spread property insurance in US on bigger geography with less cats, write more P/C instead of life insurance in Canada).

Exposure to counterparties:

It is troubling that company purchases all reinsurance from one reinsurer; also that it has a strong relationship with only one global capital player.

In stress scenarios, the reinsurer or global capital player may both encounter difficulties of their own & be unable to fulfill obligations to the insurer, so that previously expected funding sources are no longer able to be drawn on. Company should thus diversify reinsurance exposure & also develop strong relationships with other global capital market players. It should also regularly gauge ability to raise funds from each of these sources, to ensure estimates of fund raising capacity remain valid.

Contingency plans:

The CRO sets liquidity risk thresholds quarterly, but he does not develop contingency plans to be put into effect if these thresholds are breached, reducing their usefulness.

Contingency plans should thus be established (list of order of asset sales, etc.) along with the thresholds to ensure that company has credible plan of action to raise liquidity if thresholds are breached. These should be formed with input from results of stress scenarios.

Examiner's Comments

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Candidates seemed to struggle a bit with this question. In many cases, candidates identified and described four sources of liquidity risk for the sample company given in the question, but failed to mention any mitigation techniques. In other cases, candidates confused other sources of business or insurance company risk for liquidity risk.

A set of eight different identified liquidity risks and associated descriptions were used in the scoring for this question. A set of 17+ different liquidity risk mitigation techniques were identified.

Graders were careful to not allow related insurance or business risks to be awarded for credit – only sources of liquidity risk were accepted as answers. For example, while rapid growth or reserve adequacy can be solvency risks or capital adequacy risks for insurance companies, they are not explicitly liquidity risks. Another example is that many candidates mentioned the company faces catastrophe risks but failed to elaborate on how catastrophe risks can lead to liquidity risks. In some cases, a mitigation technique was given that didn't fit with the identified

and described sources of liquidity risk. In other cases, the same mitigation technique was given multiple times for different sources of liquidity risk but without further elaborating on how this same technique can be tailored to each unique source of liquidity risk. In addition, when a mitigation technique was mentioned which the company was already doing, no credit was awarded.

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Question 8:

Part A:

Model Answer: The analyst can manufacture a range of securities with different cash flow risks, or tranches, against the portfolio of high-risk subprime mortgages. These tranches should be prioritized in how they absorb losses and how they are paid from the high-risk subprime mortgages. If no one defaults on the mortgages, all tranches will be paid. However, when monthly mortgage payments are not made, payments may not reach holders of lower-rated tranches (junior tranches) since the higher-rated tranches (senior tranches) are paid first. Junior tranches, being risky, will have low prices and high promised returns, while the senior tranches, being relatively safe, will have relatively higher prices and lower promised returns.

The rating of the security is based on the anticipated likelihood of observing a default, or it is based on the expected economic loss. The assumptions are that these high-risk subprime mortgages are not correlated with each other and that they are not correlated with the market as a whole. Since the senior tranches in the proposed structure are safeguarded by the junior tranches, as long as they are not correlated, the likelihood of observing a default in the senior tranches could be low enough to qualify for the AAA rating. As correlation increases, the differentiation between the tranches diminishes. With perfect correlation, the tranching strategy produces no differentiation in risk between the tranches.

Alternate response #1: The analyst can manufacture a range of securities with different cash flow risks, or tranches, against the portfolio of high-risk subprime mortgages. For example, from a portfolio consisting of three mortgages, the analyst could construct three tranches as follows:

Senior tranche: only defaults if all three mortgages default

Mezzanine tranche: defaults if two or more mortgages default

Junior tranche: defaults if one or more mortgages default

If we assume the probability of default for each mortgage is 10% and the mortgages are independent, the probabilities of default for each default would be as follows:

Senior tranche: $(0.10)^3 = 0.001$ or 0.1%

Mezzanine tranche: $3*(0.1)^2*(0.9) + (0.10)^3 = 0.028$ or 2.8%

Junior tranche: $3*(0.9)^2*(0.1) + 3*(0.1)^2*(0.9) + (0.10)^3 = 0.271$ or 27.1%

Alternatively: $1 - (0.9)^3 = 0.271$ or 27.1%

As demonstrated above, the Junior tranche, being the most risky, will have low prices and high promised returns, while the senior tranches, being relatively safe, will have relatively higher prices and lower promised returns.

The rating of the security is based on the anticipated likelihood of observing a default, or it is based on the expected economic loss. The assumptions are that these high-risk subprime mortgages are not correlated with each other and that they are not correlated with the market as a whole. Since the senior tranches in the proposed structure are safeguarded by the junior tranches, as long as they are not correlated, the likelihood of observing a default in the senior tranches could be low enough to qualify for the AAA rating. As correlation increases, the differentiation between the tranches diminishes. With perfect correlation, the tranching strategy produces no differentiation in risk between the tranches.

Alternate response #2: The analyst can manufacture a range of securities with different cash flow risks, or tranches, against the portfolio of high-risk subprime mortgages. These tranches should be prioritized in how they absorb losses and how they are paid from the high-risk subprime mortgages. If no one defaults on the mortgages, all tranches will be paid. However, when monthly mortgage payments are not made, payments may not reach holders of lower-rated tranches (junior tranches) since the higher-rated tranches (senior tranches) are paid first. Junior tranches, being risky, will have low prices and high promised returns, while the senior tranches, being relatively safe, will have relatively higher prices and lower promised returns.

The rating of the security is based on the anticipated likelihood of observing a default, or it is based on the expected economic loss. The assumptions are that these high-risk subprime mortgages are not correlated with each other and that they are not correlated with the market as a whole. Since the senior tranches in the proposed structure are safeguarded by the junior tranches, as long as they are not correlated, the likelihood of observing a default in the senior tranches could be low enough to qualify for the AAA rating.

Furthermore, the analyst can manufacture additional securities by combining the more junior tranches from several pools of mortgages into additional securities with different tranches, often called a CDO². By defining a senior tranche in the CDO² that only defaults when multiple junior tranches default, the analyst can manufacture additional low risk securities which may have default probabilities that could be low enough to qualify for the AAA rating.

As correlation increases, the differentiation between the tranches diminishes. With perfect correlation, the tranching strategy produces no differentiation in risk between the tranches. The CDO² structure magnifies the impact of correlation.

Part B:

Structured Finance Vehicles artificially created a supply of credit at very low cost to the market. The demand for these vehicles was very high due to their perceived low risk (AAA-rated) and attractive yields compared to similar-rated investments. Also, the role of federally sponsored agencies (Fannie/Freddie) in packaging CDOs for sale to the markets led to the assumption of an implied federal backstop or air of legitimacy.

CDOs allowed banks to be isolated from credit risk, because they could simply transfer the credit risk of the mortgages they were writing to investors. Hence, they had incentives to reduce underwriting standards in order to generate more loans and satisfy the demand of the market.

Unfortunately, many stakeholders did not understand the risk inherent in CDOs. Rating agencies failed to recognize how sensitive the instruments were to assumptions about default probabilities, recovery values, correlation of defaults between each other and correlation with the market (systematic risk).

- Default probabilities were higher because of lower quality borrowers
- Correlation/systematic risk was higher due to bundling of mortgages from similar geographic areas and credit segments.
- Recovery rates were lower than expected because assets had to be sold under pressure (fire sales)

CDOs by their nature amplified the above problems. Also, synthetic CDOs (credit default swaps) artificially inflated the bubble as CDOs could be mimicked without the actual underlying assets.

Default rates were also thought to be low because it was assumed that housing prices would rise continuously (based on recent history). As a result, borrowers would be able to either refinance or sell their homes for a higher price than the mortgage, reducing the risk of default.

However, when the housing bubble burst, home prices decreased and many owners defaulted. This caused junior and mezzanine tranches to default, but losses also made their way to senior tranches that were perceived to be risk-free. Consequently, these tranches had their credit ratings downgraded which led to mark-to-market losses. They were often used as collateral, so calls for more collateral were made to compensate for the decreased value. At this point, funding in the market was scarce as banks got caught warehousing subprime mortgages to be repackaged, but there was no longer a market for them. A liquidity crisis ensued. A fire-sale of assets (not necessarily related to the housing market) was necessary in order to meet collateral requirements.

(Note: This model answer captures all of the components for which credit was given. To receive full marks, a candidate was required to cite five of the concepts listed above)

Part C:

Acceptable reasons why credit rating agencies ARE to blame for the erosion of structured finance:

- 1) Rated/Evaluated CDOs and deemed them “safe”
- 2) Rating agencies used the same scale as single name (corporate bond) ratings, which was misleading to investors
- 3) Rating agencies failed to appreciate the fragility of their models, especially correlation/systematic risk
- 4) Rating agencies did not consider systematic risk, which led to mispricing and enhanced demand especially for more Junior tranches

- 5) Rating agencies have inherent conflicts of interest because issuer pays for rating
- 6) Rating agencies failed to consider the risk/impact decreasing housing prices.
- 7) Rating agencies failed to consider the risk/impact decreasing underwriting standards.

Acceptable reasons why credit rating agencies ARE NOT to blame for the erosion of structured finance:

- 1) Regulators & Investors bought into the ratings – they did not do their own due diligence / did not seek to validate or understand risks
- 2) Regulators tied bank capital requirements to ratings
- 3) Investors did not consider whether ratings meant the same thing for CDOs as they did for corporate bonds
- 4) Originating banks became fee-driven, not concerned with underwriting risk as long as the mortgages could be sold
- 5) Investors only focused on ratings – were not concerned that little to no underwriting was being done by originators
- 6) Investment banks & rating agencies had perverse incentives to keep the machine running
- 7) Home buyers overextended themselves

(Note: This model answer captures all of the components for which credit was given. To receive full marks, a candidate was required to cite three of the reasons listed above)

Examiner's Comments:

Part A:

Tranching: Most candidates identified tranching as the first step. Many candidates were able to explain the prioritization of cash flows to the various tranches.

Structuring: Candidates were somewhat less articulate in describing how structuring creates tranches with varying degrees of risk. Many candidate got at least some credit for this section though.

Correlation / Independence: Many candidates mentioned low correlation or independence. Those using an example often stated an independence assumption. Fewer candidates were able to articulate the link between low correlation to successful tranching.

Common point deductions were for simple math errors where examples were used, incomplete descriptions of structuring, and mentioning correlation without linking it to successful tranching.

Some candidates also included discussion of prepayment risk in addition to default risk. Although our grading rubric focused on default risk, we did not penalize candidates for including a discussion of prepayment risk.

Part B:

The quality of candidate responses on Part B varied widely. Most candidates were able to describe at least a few of the various factors that led up to the financial crisis. Some candidates took a fairly deep dive into one particular item, which was a less successful strategy. We used a fairly lengthy list of possible acceptable responses, and were generally fairly liberal on grading this section.

A significant amount of candidates focused on the chain of events that happened after the housing bubble burst, rather than focus on why these investments were flawed and misunderstood or how they fueled the housing bubble. We felt that these responses were acceptable and adjusted our rubric accordingly.

Part C:

Many candidates decided to take a side and list reasons why rating agencies were or were not to blame. A completely acceptable, and often more successful, alternative strategy would be to list a couple of reasons from both columns, but this was a less common strategy. A few candidates combined their responses to B & C, or referenced the part B response in Part C. In this case, we sought to give credit to valid responses.

Question 9:

Part a.

Solution 1

1. Consistent- metric is dollars of loss, so comparison can be made across, for instance, business units.
2. Probability based- since it's based on probability, the user can set the confidence to some level and then use VaR as a prospective view of risk
3. Common time horizon- VaR can be set to use common time horizons, which makes comparisons easier

Solution 2

1. It's easy to calculate
2. It's simple to understand
3. It's easy to compare against other categories/lines of business

Solution 3

1. VaR is a common dollar measure that can be used to compare risks across lines and companies
2. VaR is probability based so it can be used to develop a result at any level of confidence
3. Based on the normal distribution, so the calculations are understandable and straightforward

Solution 4

1. Could be used across different lines to compare risks
2. Provides a number to be x% sure that the liabilities will be within a threshold
3. Is widely used and accepted industry-wide

Solution 5

1. VaR allows for same % measure to be used across lines of business so as to provide an easy comparison among risks/capital needs of lines in an equivalent manner.
2. VaR is a relatively easy and straightforward method to understand. People understand that there is an x% chance of a loss of \$N.
3. VaR enables capital to be allocated to various lines based on the risk level at that threshold.

Part b: Process

Solution 1

VaR can be used to calculate an exceedence probability. This is the probability that actual loss will be greater than expected loss plus allocated capital. To allocate capital, you allocate it such that each line has the same exceedence probability.

Solution 2

To do this, select a % probability of ruin that is relevant. Let's say 1%. Calculate the 99% (1-.01) VaR for each line of business and allocate proportionally to that.

Example:	99% VaR	%	Allocated Capital
Line A	1M	1/3	6M x 1/3 = 2M
Line B	2M	2/3	6M x 2/3 = 4M
Total	3M		6M

*assume total capital chosen was 6M

Solution 3

Get the distributions that describe the lines of business' losses. Select a percentage (say 99% if you want losses to be exceeded 1% of the time) for each line. Determine the value of loss associated with each percentage. Determine an aggregate (all line) distribution. Determine the proportion of each line's value to the aggregate.

Part b: Issues

Solution 1

1. There may not be enough capital to allocate so all lines have the target exceedence probability
2. This approach does not consider the benefit of diversification among lines of business
3. Assumes a normal distribution when in reality a heavier tailed distribution might be more appropriate

Solution 2

1. The firm may not have enough capital to achieve a specific exceedence probability
2. It does not reflect the impact of diversification across lines
3. VaR is one sided: reflects the downside of the loss but the potential upside is not reflected

Solution 3

1. This does not consider severity of the losses exceeding the threshold. If this is an important consideration, he should use another measure, such as CTE.
2. There may not be enough capital to hit the desired exceedence probability for all lines.

3. Diversification benefits between lines are not considered

Part b: Risk Horizon

Solution 1

Motivation- it is important to choose an appropriate time horizon because assets and liabilities change with time and so does the risk capital.

Consideration- The selection of a time horizon can be subject to what regulators have asked for (usually 1 year).

Solution 2

Should be a short risk horizon. VaR does not work well over long time periods because 1) insufficient data for empirical analysis and 2) the tail of the normal distribution is too thin.

Solution 3

A motivation to select a risk horizon would be to align with regulators/rating agency reviews so that it ties to market consequences like downgrades or regulatory action. A consideration is that it's hard to validate long time horizons due to lack of sample size of independent time bands.

Solution 4

If extended too far, the risks may change over time. Further, data is sparse as time intervals lengthen. Often times, 1 year is an adequate interval for availability of data, composition of portfolio doesn't change drastically, and it aligns with annual statements for the company.

Examiner's report:

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The question was meant to test the students' understanding of VaR, how VaR can be used in the capital allocation process, and an understanding of how to choose a risk horizon.

**Part a**

The advantages were generally well answered.

**Part b, process**

Most candidates were able to sufficiently describe a process to use VaR to allocate capital. If a candidate's response included the use of exceedence probabilities, the candidate only received full credit if he/she defined "exceedence probability".

**Part b, issues**

The issues were generally well answered. Most candidates were able to successfully list two of the three issues (derived from multiple sources) while about one-third of the pool was able to list all three.

**Part b, risk horizon**

This was the part of the question that had the most errors, in large part due to the confusion as to what was expected from the candidate for this portion. In this part, the candidates were expected to list two things: a motivation and a consideration. Candidates did not distinguish between a consideration and motivation of selecting risk horizons. Thus full credit was given if candidates could successfully list two items (regardless of if they were considerations or motivations) regarding risk horizon selection. Most candidates did not receive full credit on this part of the question.

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Question 10:

a)

<u>Risk</u>	<u>Var</u>	<u>% of Total</u>	<u>Allocated</u>
U/W	100M	40%	$40\% * 180 = 72M$
Res	80M	32%	
Credit	70M	28%	
Total	250M	100%	180M

b)

<u>Risk</u>	<u>Total</u>	<u>Without</u>	<u>Incremental</u>	<u>%</u>	<u>Allocated</u>
U/W	180M	140M	40M	4/15	
Res	180M	130M	50M	5/15	$180 * 5/15 = 60M$
Credit	180M	120M	60M	6/15	
Total	180M		150M		

c)

Accepted responses for justifications for risk capital:

- Book value of capital is distorted by accounting issues such as goodwill, holding bonds at amortized values and thus not reflective of the risk profile.
- Actual value of capital may not reflect the risk level that the insurer faces. It is often driven by regulatory requirements (RBC) and it often exceeds capital based on risk measures.
- Risk capital includes impact of loss reserve discounting.
- The book value of the firm is just the accounting value of capital at one point in time. It is not a prospective looking value that is helpful in predicting performance.
- Risk capital is risk-adjusted and more appropriately reflects risk potentials in each category.
- Actual capital can be a volatile number based on past underwriting performance.
- Maintains adherence to aggregate risk measure selected.
- RAROC or EVA will allow you to determine which lines of business are profitable and actually adding value to the company
- to facilitate decision making in a regulatory environment that has risk based capital rules.
- Risk capital is more flexible and allows the insurer to choose various risk metrics to calculate as appropriate.

- Book value is based on accounting rules that don't reflect how much value of an asset can be realized in a stress scenario.

Accepted responses for justifications for book:

- This is the amount of capital the company actually has to protect against risk.
- Risk capital is highly dependent on the selected risk metric.
- The insurer might not have enough capital to allocate the entire risk capital.
- If the insurer has more capital than the estimated requirement, then some of the capital will not be allocated to any risk.
- If you are interested in return on capital figures, and risk capital is less than book value, returns will be inflated with a smaller denominator.
- Estimated numbers are harder to explain to regulators.
- Book value is from the financial statements. It's reliable and readily available.

Arguments against allocating in general:

- All capital is backing up the total risk of the company. It is indivisible.

Examiner's Comments:

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a.  
 Nearly all candidates correctly applied the proportional method to the total combined VaR in this straight-forward calculation. A very few candidates failed to recognize the correct total amount of capital being allocated.

b.  
 Most candidates correctly identified the incremental method's main issue: how a given source impacts the total risk. Candidates struggled to apply this increment relative to the other source's increments. Some candidates did correctly point out that certain methodologies intentionally leave a portion of capital unallocated.

c.  
 Candidates generally struggled with this part of the question. Many arguments were cited favoring one approach over the other that equally applied to both approaches. Many justifications were incomplete. Equally, many justifications were concise and well worded. Some candidates interpreted the question as whether or not capital should be allocated to risk sources at all. Correct arguments either for or against this allocation received credit.

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Question 11:

The words "beta of liabilities" means the "beta of liabilities regarding underwriting or insurance risks". Partial credit was given if the candidates misunderstand the "beta of liabilities" as "beta of liabilities regarding investment risks".

Part a.

$$B_E = B_A * (1 + k_1 + k_2) + B_1 * s_1 + B_2 * s_2$$

$$B_E = 0.50*[1 + (600M/1000M) + (200M/1000M)] + 0.25*(500M/1000M) + 0.35*(500M/1000M)$$

$$B_E = 1.20$$

$$R_E = R_F + B_E*(R_M - R_F)$$

$$R_E = 4\% + 1.2*(10\% - 4\%)$$

$$R_E = 11.2\%$$

Part b.

$$R_i = -k_i*R_F + B_i*(R_m - R_F)$$

$$R_1 = -(600M/1000M)*4\% + 0.25*(10\% - 4\%) = -0.9\%$$

$$R_2 = -(200M/1000M)*4\% + 0.35*(10\% - 4\%) = 1.3\%$$

$$\text{Combined Ratio} = 1 - R_i$$

$$\text{Target for Line 1} = 100.9\%$$

$$\text{Target for Line 2} = 98.7\%$$

Part c.

Full credit answers include:

1. Line of business Betas are hard to estimate
2. Research has shown that rates of return are driven by other economic factors beyond Beta. Sole reliance on CAPM would ignore these other factors.
3. CAPM only accounts for bearing systematic underwriting risk that is correlated with the market equity portfolio. Most insurers have other risks to be concerned about such as extreme events and tail risk.
4. There is no active market where liabilities are traded so it is difficult to calculate the B for liabilities.
5. CAPM ignores taxes and transaction costs; transaction costs could be material if duration of assets are short enough.
6. Beta is difficult to calculate due to correlations.
7. CAPM model does not take account of or ignores diversification benefits between different lines of business.
8. CAPM's assumption on holding period and view of the future cash flow might be different from the company's.
9. Beta of liabilities are hard to estimate.
10. CAPM is a benchmark measure, not a prospective measure.
11. CAPM may not even be adequate model for the intention above, so application here can also be questioned.
12. Beta does not capture all sources of risks.
13. CAPM is hard to verify because it calculate expected return which may be different from actual return.
14. CAPM also ignores liquidity and liquidity risk.
15. Using CAPM doesn't account for the risk load of the line. So the required return is higher from CAPM.

Examiner's comments:

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Part a:

We expect candidates to be able to use the formula on page 14 and 15 in the paper "Cummins – Allocation of Capital in the Insurance Company". The formula indicates the calculation of beta of equity. Most candidates failed to calculate this piece correctly. The majority of candidates were able to apply the basic formula of CAPM model.

We noticed that the question used the term of “beta of liability” instead of “beta of insurance risk” which is used in the paper. Fortunately, we did not see many candidates being confused with the terms being inconsistently used.

Part b:

When the paid loss, paid LAE amounts and expense-related information were not given in the exam question, there is no way to calculate the combined ratio through summing up the loss, LAE ultimates and expense ratios. Candidates should expect to use the rate of return to derive the combine ratio.

Less than half of the candidates were able to write out the formula of combined ratio = 1 - rate of return. But a few candidates were able to use the right formula as stated in the page 14 to get the correct rates of return.

Though in the paper, numerical or actual example was not given, candidates are still expected to apply the formula to solve problems.

Part c:

The paper lists a few weakness of CAPM being applied in the insurance industry. Most candidates were able to answer the two of those listed weaknesses. Some candidates answered this question using general weaknesses in CAPM model. For example, the real market return rate cannot be observed or CAPM assumptions are often not realistic. Partial credit was given to these answers.

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Question 12:

Part a) One of the following would receive full credit:

- The proportion of uninvested assets (such as agents balance or premium receivables) originating from the insurance operations will tend to rise
- There would be an increase of risk to owner’s equity which would warrant a more conservative investment policy.

Part c)

Due to an increase in underwriting results a more aggressive investment policy could be pursued.

Examiner's comments:

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Part a:

This question was straightforward and majority of candidates received full credit. Most deductions occurred due to candidates writing extraneous incorrect information.

Part b:

This question part was declared invalid. No points were given for any response and was taken out of consideration of the pass mark.

Part c:



*This question was straightforward and majority of candidates received full credit. A common error was merely stating the company had more money to invest but did not state how the company would invest it.*

**Question 13:**

*Part a:*

$$y = \text{ROE} = 0.15 \text{ and } z = \text{standard normal desired level} = \text{Normsinv}(.98) = 2.054$$

$$\text{Risk Load Multiplier} = \lambda = y / (1 + y) * z = 0.26788$$

$$\text{Var}(x) = \sum p(x) * (1 - p(x)) * x^2$$

$$\text{Marginal St Dev A} = \text{Var}(A+B)^{0.5} - \text{Var}(B)^{0.5} = 696.82^{0.5} - 105.3^{0.5} = 16.13$$

$$\text{Marginal St Dev B} = \text{Var}(A+B)^{0.5} - \text{Var}(A)^{0.5} = 696.82^{0.5} - 264.4^{0.5} = 10.13$$

$$\text{Risk load} = \lambda * \text{marginal st dev} \rightarrow A = 4.322, B = 2.715$$

*Part b:*

Company should expect to underachieve target return. The MS method underallocates risk loads due to sub-additive nature of standard deviations.

*Alternate:* Company should not expect to achieve target return as the sum of the individual risk loads is less than the total required for both accounts combined due to the sub-additive nature of square roots.

*Part c:*

Company should expect to over-achieve target return as the MV method double-counts covariance, so risk load is too high, and company would overcharge premium.

*Part d:*

Higher market risk margin means higher ROE target, which will increase risk loads.

*Examiner's comments:*

*Part a:*

*Most candidates did well. Common errors included incorrect z-score (typo of 2.54 rather than 2.054) and incorrect variance formula ( $E[x^2] - E[x]^2$  using total account values than for each event and summing).*

*Part b:*

*Most candidates did well. Common mistakes: lower risk load means lower capital so better ROE, didn't explain why risk load is too low (square root or standard deviation).*

*Part c:*

*Candidates did well on this part of the question. Common mistakes were candidates wrote premium/risk load is higher without stating relative to what. Also some forgot to mention the cause of overcharge (double charge of covariance).*

*Part d:*

*Some candidates neglected to say ROE goes up (or down with negative beta).*

**Question 14:**

*Part A:*

*Model Solution 1:*

Ruin theory is concerned with solvency. CAPM says that it is not enough to simply remain solvent, a return must be earned for investors as well.

Model Solution 2:

Ruin theory requires:

$$P(L+E > E(L)+E+R+V) < e$$

R = return

V = surplus

E = exceedence prob

L = loss

E = expense

CAPM modified this requirement by stating it is not enough to break even, you must also earn a return. In other words, CAPM requires:

$$P(L+E+R > E(L)+E+R+V) < e$$

Model Solution 3:

Ruin theory says you need to hold enough so that you are not bankrupt some percentage of time. CAPM takes it one step further, says you need to hold enough capital and make a certain return on that capital to investors and so you can't count the expected return (R) in your initial capital.

*Part B:*

Model Solution 1:

Using historical loss experience to estimate S may prove insufficient if:

- i) The reserving adequacy has changed
- ii) The mix of business has changed

Model Solution 2:

Risks that are not associated with the loss distribution such as:

- i) Investment risk – risk that assets will not earn their expected returns
- ii) Default risk – risk that reinsurers will not be able to pay recoveries

Model Solution 3:

- i. Insurance loss distribution are non-normal so std dev may not be an accurate barometer of risk (need to consider fat tails and extreme events).
- ii. Business growth/decline could make the past estimator of S unreliable since it's based on a different mix of business.

*Examiner's comments:*

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For Part A most candidates did not score well on this part because they failed to communicate clearly by discussing the two approaches and clarifying the difference. Many candidates defined ruin theory within its own terms (i.e. ruin theory is focused on the probability of ruin)

For Part B there were a wide variety of answers that received full credit (a subset of which is provided in the model answers). The majority of candidates focused on why past loss experience may not be a good predictor of an insurance operation's variability. Several candidates identified other types of risks (i.e. regulatory risk, operational risk, etc.), but did not provide a description of the risk so they only received partial credit for that portion. Additionally, if a candidate stated that there are other risks beyond insurance contract risks, but did not name any of those risks, they did not receive any credit.
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**Question 15:**

Part a.

Given  $F = \$4M$ ,  $y = 0.12$ ,  $r_f = 0.02$ ,  $m_L = \$3M$

$$R = A(y - r_f)/(1 + r_f) \quad (\text{risk load formula})$$

$$P = R + m_L / (1 + r_f) \quad (\text{premium formula})$$

$$F = P + A = R + m_L / (1 + r_f) + A = A(y - r_f)/(1 + r_f) + m_L / (1 + r_f) + A$$

$$\$4M = A(.12 - .02)/(1.02) + \$3M / (1.02) + A$$

$$A = \$964,285.71$$

$$R = \$94,537.82$$

$$P = \mathbf{\$3,035,714.29}$$

.25 each for correct use of (F, y)

.25 for correct risk load formula

.25 for correct premium formula

-.25 points for a mathematical error

Alternative solution 1

$$(1+y)A = (1+r_f)F - m_L$$

$$1.12A = 1.02(\$4M) - \$3M$$

$$A = \$964,285.71$$

$$R = \$94,537.82$$

$$P = \mathbf{\$3,035,714.29}$$

Alternative solution 2

$$(1+r_f)F \geq s$$

$$(1.02)\$4M \geq s$$

$$s \leq \$4.08M$$

$$A \geq (s - m_L) / (1+y)$$

$$A \geq (\$4.08M - \$3M) / (1.12)$$

$$A = \$964,285.71$$

$$R = A(y - r_f) / (1 + r_f)$$

$$R = \$964,285.71(.12 - .02) / (1.02)$$

$$R = \$94,537.82$$

$$P = R + m_L / (1 + r_f)$$

$$P = \$94,537.82 + \$3M / (1.02)$$

$$P = \mathbf{\$3,035,714.29}$$

Part b.

Possible answers for what the loss safety level represents: (0.5 pts)

- It is a measure of reinsurer company conservatism.
- It is the minimum amount of funds the reinsurer wants to have in order to cover losses when they occur at predetermined target percentile

Possible answers for the implications of a higher loss safety level: (0.25 pt)

- Higher safety level will result in more expensive and less competitive contracts
- Reinsurer needs to allocate more funds
- The higher the safety level, the less probable that the safety level will be exceeded
- Higher safety level will require higher risk loads

Part c.

A stochastic model needs to be run iteratively until the safety constraint (determined by the tail of the stochastic distribution) and the rate of return (mean return of stochastic distribution/equity) are both met. A possible process

1. Build a simulation model using the inputs given in the problem, translating the given mean and standard deviation into a loss distribution.
2. Calculate a starting value for the risk load, based on a defined safety level, and run the model
3. Determine if the variance constraint is satisfied and add surplus if it is not.
4. Repeat until both constraints are satisfied.

*Examiner's comments:*

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Part A – Most candidates receiving full credit recognized the need to first solve for the assets allocated by the reinsurer. A common mistake was to assume that the \$4 million provided in the question represented the safety level or the allocated assets. Another common mistake was to use the yield rate of the current investment portfolio (5%), as opposed to the target return on equity (12%), for the yield rate in the asset and risk load formulas. Most candidates performed well when it came to applying the risk load formula and premium formula. Some candidates provided alternative solutions which we include in the model solution.

Part B – Most candidates performed very well on this question. We accepted a number of different responses as shown in the model solution. Responses that did not receive full credit often did not include an implication of having a higher loss safety level, or simply provided a formula for the loss safety level without any corresponding

explanation. Some candidates incorrectly described the safety level as a present value or “current value” and not as the final (or accumulated) value that needs to exist at the time of loss.

Part C – It was determined after the exam sitting that the topic addressed by this question is not on the syllabus for Exam 9. Candidates who left the question blank were not negatively impacted, but those candidates who took the time to provide a response were given credit where appropriate. Most candidates who received partial credit on this question recognized the need for a simulation procedure and provided some explanation around how such a procedure might be set up and run. Responses that simply restated the Kreps procedure without providing a sufficient explanation about how this would be impacted by allowing for multiple payments were not awarded credit for this question.

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**Question 16:**

A. Company A, Company B and Company C are all writing business in the same market, yet the rates for Company C are higher than either of its competitors.

The ROE or return on capital that Company C is earning is several percentage points higher than either of its competitors and over the regulator’s threshold of 15%. Therefore Company C’s rates are excessive.

|   | L&E | Prem | Profit | Capital | Ret. on Cap. |
|---|-----|------|--------|---------|--------------|
| A | 295 | 320  | 25     | 213     | 11.7%        |
| B | 310 | 330  | 20     | 147     | 13.6%        |
| C | 330 | 345  | 15     | 86      | 17.4%        |

Profit = Prem – L&E

Capital = Prem/(Premium to Capital Ratio)

Return on Capital (ROE) = Profit/Capital

B. Even though all three companies write business in the same market, Company C’s business is expected to produce higher losses and expenses than its competitors. In addition, Company C’s return on capital appears inflated because of the high ratio of premium to capital that the company operates at. Need one of the following 3 options for full credit.

Option 1: Company C’s profit margin, as a percentage of the premium rate, is lower than either of its competitors. Therefore Company C’s rates are not excessive.

|   | L&E | Prem | Profit \$ | Profit % |
|---|-----|------|-----------|----------|
| A | 295 | 320  | 25        | 7.8%     |
| B | 310 | 330  | 20        | 6.1%     |
| C | 330 | 345  | 15        | 4.3%     |

Profit = Prem – L&E

Profit % (ROS) = Profit/Prem

Option 2: Company C's combined ratio (1- profit %) is higher than either of its competitors. Therefore Company C's rates are not excessive.

|   | L&E | Prem | CR    |
|---|-----|------|-------|
| A | 295 | 320  | 92.2% |
| B | 310 | 330  | 93.9% |
| C | 330 | 345  | 95.7% |

CR = (L&E)/Prem

Option 3: Company C's markup is lower than either of its competitors. Therefore Company C's rates are not excessive.

|   | L&E | Prem | Markup |
|---|-----|------|--------|
| A | 295 | 320  | 8.5%   |
| B | 310 | 330  | 6.5%   |
| C | 330 | 345  | 4.5%   |

Markup: Profit/(L&E) - 1

C. Need 2 of the following 3 for full credit:

1. Investment Income on surplus: "While it provides protection to policyholders and claimants, the surplus does not belong to them. In fact, the inclusion of investment income on surplus creates a situation in which an insurer with a large surplus relative to premium must charge lower rates than an otherwise equivalent insurer with less surplus. In other words, lower cost for more protection."
2. Investment Income in excess of the risk free rate: "The existence of an opportunity cost does not give the policyholder claim on some part of the actual earnings of the insurer. Should the insurer engage in speculative investments resulting in the loss of policyholder supplied funds, the company cannot assess the insureds to make up the shortfall."
3. Investment Income from 100% of policy cash flows: Only a portion of cash flows generate investment gains. "Not all cash flows go through invested assets – some portion being required for the infrastructure of the insurer."

*Examiner's comments:*

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a.) Most candidates did very well calculating the return-on-capital of the 3 companies and drawing the conclusion that C's rates could be viewed as excessive because they had the highest ROC (or ROE) in the market. A number of candidates only pointed out that C had the highest rates of the 3 companies, and while this is certainly true, it was not a robust enough answer for full credit.

b.) Most candidates also did very well calculating the return-on-sales of the 3 companies and drawing the conclusion that company C could defend their rates as not being excessive because they had the lowest return-

on-sales (or underwriting profit provision) in the market. We also gave full credit for comparing combined ratios or other profitability measures (such as markup over losses) with whatever synonym the candidate chose, as long as the label correctly matched the calculations. Like part A, a good number of candidates just pointed out that company C had the highest losses, or described in words that their ROC in part A was inflated due to high leverage, but that was not robust enough for full credit. Similarly, candidates who compared only nominal dollars of profit received substantial partial credit but not full credit.

c.) The 3 most common answers for this part came from the McClenahan paper. The most common reasons that candidates did not get full credit for each one identified are listed below, separated by each type of investment income.

- i.) Investment income on PHSF in excess of the risk-free rate: Many candidates described this one very well and got full credit. The most common reason to lose some credit was not to be clear that only the *excess* would be excluded, or to confuse taking on investment risk with other insurance risk.
- ii.) Investment income on surplus: Again, many candidates described this one well and received full credit, and we accepted various alternatives to “surplus” such as capital, shareholder supplied funds, etc. The most common pitfall in the description was to be overly brief and jot down something along the lines of “surplus doesn’t belong to them.” This received partial credit, but for full credit we were looking for a more complete short discussion of the issue. Many candidates correctly added on that this would penalize well-capitalized insurers. Within that second assessment, a common error was to confuse the terms “highly capitalized” with “highly leveraged”, which are in fact opposites.
- iii.) Investment Income from 100% of policy cash flows, or, investment income from the portion of premium set aside for infrastructure: Most candidates who chose this answer were able to identify that infrastructure expenses should be excluded in some way, but relatively few made the connection that a key reason for the exclusion is that the funds aren’t investible (i.e. no investment income) and thus lost some credit.

Question 17:

Part A Model Solution 1:

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.025P				.025P
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(P - 50 - 0.5P) - 0.2P/1.1^{.25} - 0.35P/1.1^{.5} + 0.025P/1.1 - 0.25P/1.1^{1.5} + 0.5P/1.1^2 + 0.025P/1.1^2 = 0$$

P = 237.06 assuming no reserves set up

Part A Model Solution 2:

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.025P				.025P
Reserves (L & V)	-0.8P	0.2P	0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(-0.3P - 50) + 0.025P/1.1 + 0.5P/1.1^2 + 0.025P/1.1^2 = 0$$

P = 319.26 assuming reserves set up for Losses & Variable Expense

Part A Model Solution 3:

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.025P				.025P
Reserves (L)	-0.6P		0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(-0.1P - 50) - 0.2P/1.1^{.25} + 0.025P/1.1 + 0.5P/1.1^2 + 0.025P/1.1^2 = 0$$

P = 309.94 assuming reserves set up for Losses

Part B.i Model Solutions:

1. Reduce fixed expenses
2. Reduce variable expenses
3. Delay paying fixed expenses
4. Delay paying variable expenses
5. Improve underwriting and pay less losses
6. Delay paying losses
7. Allocate less capital to this line
8. Release capital earlier once losses & expenses fully paid
9. Allocate capital based on reserves and release as reserves are paid
10. Invest more aggressively and get better returns
11. Invest other assets as well

Part B.ii Model Solutions:

Due to the numerous possible solutions to Part B.i, not all full credit solutions can be listed here but full credit was given if implemented 100% correctly. Candidates who implemented modifications on other model solutions also received full credit if done 100% correctly.

Model Solutions shown for P = 237.06 in Part A

Invest in portfolio that yields 10%

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.05P				.05P
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(P - 50 - 0.5P) - 0.2P/1.1^{.25} - 0.35P/1.1^{.5} + 0.05P/1.1 - 0.25P/1.1^{1.5} + 0.5P/1.1^2 + 0.05P/1.1^2 = 0$$

$$95\% * 237.06 = 225.21$$

$$P = 196.62 < 225.21$$

Candidates who solved for the investment return of 6.28% that would make the premium exactly 225.21 also received full credit.

Assume a 4:1 premium to surplus ratio

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.25P								0.25P
Inv Inc					.0125P				.0125P
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(P - 50 - 0.25P) - 0.2P/1.1^{.25} - 0.35P/1.1^{.5} + 0.0125P/1.1 - 0.25P/1.1^{1.5} + 0.25P/1.1^2 + 0.0125P/1.1^2 = 0$$

$$95\% * 237.06 = 225.21$$

$$P = 214.95 < 225.21$$

Candidates who solved for the P:S ratio of 2.687 that would make the premium exactly 225.21 also received full credit.

Model Solutions shown for P = 309.94 in Part A

Reduce variable expenses to 15% of premium

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.15P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.025P				.025P
Reserves (L)	-0.6P		0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(-0.1P - 50) - 0.15P/1.1^{.25} + 0.025P/1.1 + 0.5P/1.1^2 + 0.025P/1.1^2 = 0$$

$$95\% * 309.94 = 294.44$$

$$P = 237.93 < 294.44$$

Candidates who solved for the variable expense % of 19.1 that would make the premium exactly 294.44 also received full credit.

Reduce fixed expenses to \$45

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-45								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.025P				.025P
Reserves (L)	-0.6P		0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(-0.1P - 45) - 0.2P/1.1^{.25} + 0.025P/1.1 + 0.5P/1.1^2 + 0.025P/1.1^2 = 0$$

$$95\% * 309.94 = 294.44$$

$$P = 278.95 < 294.44$$

Candidates who solved for the fixed expense of 47.5 that would make the premium exactly 294.44 also received full credit.

Model Solutions shown for P = 319.26 in Part A

Invest reserves as well as capital

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P								0.5P
Inv Inc					.04875P				.03125P
Reserves (L & V)	-0.8P	0.2P	0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

T = 1

Capital invested for 12 months = 0.025P

Variable expense invested for 3 months = 0.2P * 0.05 * 0.25 = 0.0025P

Loss #1 invested for 6 months = 0.35P * 0.05 * 0.5 = 0.00875P

Loss #2 invested for 12 months = 0.25P * 0.05 = 0.0125P

T = 2

Capital invested for 12 months = 0.025P

Loss #1 invested for 6 months = 0.25P * 0.05 * 0.5 = 0.00625P

$$(-0.3P - 50) + 0.04875P/1.1 + 0.5P/1.1^2 + 0.03125P/1.1^2 = 0$$

$$95\% * 309.94 = 303.30$$

$$P = 272.68 < 303.30$$

Release capital at time 1.5 once all losses and expenses are fully paid. Investment income still earned at time 2 as stated in problem.

Time	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
Premium	P								
Fixed	-50.00								
Variable		-0.2P							
Loss			-.35P				-.25P		
Capital	-0.5P						0.5P		
Inv Inc					.025P				.0125P
Reserves (L & V)	-0.8P	0.2P	0.35P				0.25P		
PV Factor	1.000	0.976	0.953	0.931	0.909	0.888	0.867	0.846	0.826

$$(-0.3P - 50) + 0.025P/1.1 + 0.5P/1.1^{1.5} + 0.0125P/1.1^2 = 0$$

$$95\% * 309.94 = 303.30$$

$$P = 300.39 < 303.30$$

Examiner's comments:

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The below were common mistakes found in each part of this question:

Part A:

1. Grouping cash-flows from different time periods into 1 time period
2. Earning investment income too often or on assets that should not have been investable
3. Mistiming various equity flows or giving various ones the wrong sign
4. Releasing capital multiple times
5. Earning UEPR unevenly or not holding enough reserves to pay for the t = 1.5 loss

Part B:

1. Not reflecting investment income impact if changed capital to surplus ratio
2. Earning investment income on reserves for the whole period even though they are paid out before
3. Lowering the premium to surplus ratio instead of increasing it
4. Having solutions greater than 95% of the Part A solution or even higher
5. Work that did not correlate to the modification described
6. Many candidates did not have enough time to do the calculations for the modifications
7. Many candidates did not do the math or setup correctly, trying to do harder calculations (ie. calculate the exact the investment income that would result in exactly 5% less premium) rather than make simpler modifications (ie. lower fixed expenses)

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Question 18:

Model Solution #1 (modeled after candidate 212)

PV CF Return → Set PV(Total Cash Flow, use discount rate) = PV(Change in Equity, use target return)

$$U = 1 - CR = 0.055 = 1 - (L+E)/P = 1 - ((75+10+0.24P)/P)$$

$$0.945 = (85+0.24P)/P \rightarrow 0.945P = 85 + 0.24P, P = 120.57 \rightarrow \text{Surplus} = P/2 = 60.29$$

$$VE = 0.24P = 28.44$$

$$PV(\text{TCF}) = PV(\text{U/W CF} + \text{Inv Inc} - \text{Tax})$$

U/W CF

t	Pd Prem	Pd Exp	Pd Loss	CF
0	120.57	10+28.94 = 38.94	0	81.63
.25			75(.35) = 26.75	-26.25
.5			75(.25) = 18.75	-18.75
.75			75(.25) = 18.75	-18.75
1.0			75(.15) = 11.25	-11.25

Inv Inc

t	Surplus (S)	Inv Inc	Total	Tax	Disc	PV(TCF)
0	60.29	0	81.63	81.63(.34) = 27.75	1.08 ⁰ = 1.00	53.88
.25	60.29	S * (.1/4) = 1.51	-24.74	-8.41	1.08 ^{-.25}	-16.02
.5	60.29	1.51	-17.24	-5.86	1.08 ^{-.5}	-10.95
.75	60.29	1.51	-17.24	-5.86	1.08 ^{-.75}	-10.74
1.0	60.29	1.51	-9.74	-3.31	1.08 ⁻¹	-5.95

$$\Sigma = 10.22$$

Δ in Equity

t	Equity = Surplus * 1.3	Δ in Equity	Disc
0	78.38	78.38	1.15 ⁰ = 1.00
.25	78.38	0	
.5	78.38	0	
.75	78.38	0	
1.0	78.38	-78.38	1.15 ⁻¹

$$78.38 * 1.00 - 78.38 * 1.15^{-1} = 10.22$$

Therefore PV(Δ in Equity) = PV(TCF), so U/W profit provision of 5.5% is consistent.

Model Solution #2

Qtr	Prem Pd	Paid Loss	Paid Expense	U/W Cash Flow (pre tax)
0	P	0	10+0.24P	.76P – 10
1	0	26.25		-26.25
2	0	18.75		-18.75
3	0	18.75		-18.75
4	0	11.25		-11.25

Qtr	Surplus	Inv Inc	Total Pre-Tax Cash Flow
0	.5P	0	.76P - 10
1	.5P	.5P * .1/4 = .0125P	.0125P – 26.25
2	.5P	.0125P	.0125P – 18.75
3	.5P	.0125P	.0125P – 18.75
4	.5P	.0125P	.0125P – 11.25

$$PV(\text{after tax cash flows}) = (1-.34) * [.76P-10+ (.0125P-26.25)/1.08^{.25})+ (.0125P-18.75)/1.08^5)+ (.0125P-18.75)/1.08^{.75})+ (.0125P-11.25)/1.08^1] = .5331P - 54.0606$$

Qtr	Equity Change
0	.65P
1	
2	
3	
4	-.65P

$$PV(\Delta \text{ in Equity}) = .65P - .65P/1.15 = .0848P$$

$$.0848P = .5331P - 54.0606$$

$$P = 120.59$$

$$U = (120.59 - 10 - .24*120.59 - 75)/120.59 = 5.51\%$$

Consistent with calculated U/W profit provision.

Model Solution #3

$$P = (75+10)/(1-.24-.055) = 120.57$$

T	Premium	Expenses	Losses	U/W CF
0	120.57	-10-.24(120.57)	-	81.63
1	-	-	75*.35	-26.25
2	-	-	75*.25	-18.75
3	-	-	75*.25	-18.75
4	-	-	75*.15	-18.75

T	Surplus	II
0	120.57/2 = 60.29	-
1	60.29	60.29*0.025 = 1.5
2	60.29	1.5
3	60.29	1.5
4	60.29	1.5

T	UW	II	Tax	CF
0	81.63	-	27.75	53.88
1	-26.25	1.5	-8.42	-16.33
2	-18.75	1.5	-5.87	-11.38
3	-18.75	1.5	-5.87	-11.38
4	-11.25	1.5	-3.32	-6.43

$$PV(CF) = 53.88 - 16.33/1.02 - 11.38/1.02^2 - 11.38/1.02^3 - 6.43/1.02^4 = 10.2681$$

T	Equity	ΔQ
0	1.3*60.29 = 78.38	78.38
1	78.38	-
2	78.38	-
3	78.38	-
4	78.38	-78.38

$$PV(\Delta Q) = 78.38 - 78.38/1.15 = 10.2235$$

Since $PV(CF) = PV(\Delta Q)$ at $U = 0.055$, this is consistent with the underwriting profit calculated by the PV CF Return Model.

Model Solution #4

Calculate Premium using $U=5.5\%$, $P = (L+E)/(1-V-U) = (75+10)/(1-.24-.055) = 120.57$

Cash Flows

T	Premium	Expense	Loss	Surplus	Inv Inc	CF w/Tax	PV w/ 8% discount rate
0	120.57	38.94	0	60.28	0	53.88	53.88
.25	0	0	26.25	60.28	$60.28(1.1)^{.25} - 60.28 = 1.45$	-16.37	-16.06
.5	0	0	18.75	60.28	1.45	-11.42	-10.97
.75	0	0	18.75	60.28	1.45	-11.42	-10.78
1	0	0	11.25	60.28	1.45	-6.47	-5.94

Sum =

10.08

Equity: $60.28(1.3) = 78.36$

PV of Δ in Equity = $78.36 - 78.36(1.15)^{-1} = 10.22$

PV of Cash Flows = PV of Δ in Equity

10.08 = 10.22

Therefore 5.5% is the appropriate UW profit.

Examiner's comments:

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Candidates were expected to write out three formulas. The easiest ways to miss the first formula,  $PV(\text{change } E) = PV(\text{TCF})$ , was either say PV of Equity rather than PV of Change in Equity, or to describe the wrong discount rates. However, if candidates wrote the formula incorrectly, but did the problem right, they were still given credit for this one. For the second formula,  $PV(\text{TCF})$ , grading was fairly lenient; if Candidates had all of the pieces,  $PV(\text{UW CF})$ , Investment Income, and Taxes in some form, credit was given. For the  $PV(\text{UW CF})$ , the most common mistake here was not taking the PV of one of the pieces.

Most candidates were able to correctly calculate the Premium and  $PV(L)$ , even more than getting the initial 3 formulae.

Quite a few students kept P (premium) until the end of the problem or used a Combined Ratio to calculate P, so we also gave credit in those circumstances.

To get the losses they just had to break the losses out by period, if a student received partial credit, it was usually from this part. For discounting, credit was given if the candidate used 1.08 (divide by  $1.08^{.25}$  or 1.02 or even 1.08 for each quarter) in some form to bring the losses back to time 0. (Unless they explicitly said, "I'm assuming everything will be compared at  $t=1$ ", this is because  $t=0$  is an explicit assumption in the paper). If they calculated their losses correctly based on their breakout and discounting, a credit was given, and most candidates got this.

For investment income credit was given as long as they had .5P (or the number) somewhere and then used it in some form or fashion to calculate their investment income. Commonly missed, the author explicitly breaks out the investment income payments by period, but the students very often just assumed a single payment at the end of

the year. The discounting of investment income was missed or gotten for the same reasons as the discounting of losses

For Taxes of UW CF and Investment Income. The most common error for Tax of UW gains was candidates applying the tax rate to the non-PV losses. Grading was very lenient with Investment Income Taxes and as long as they applied the correct tax rate to some form of investment income, candidates would get credit. In fact if they calculated an investment income and multiplied it by the tax rate and added that in rather than 1 minus the tax rate, we still gave credit for Investment Income Taxes, we just didn't give anything for Investment Income.

Candidates needed to have .65P for equity (or a calculated number for whatever they got for P), and then somehow showed that they were using it in a part-1 like formula to get credit. Candidates had to get the discounting (1.15) correct here to get credit, but this was commonly missed. Another common error here was either reversal of signs or only calculating Equity and not the Change in Equity.

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