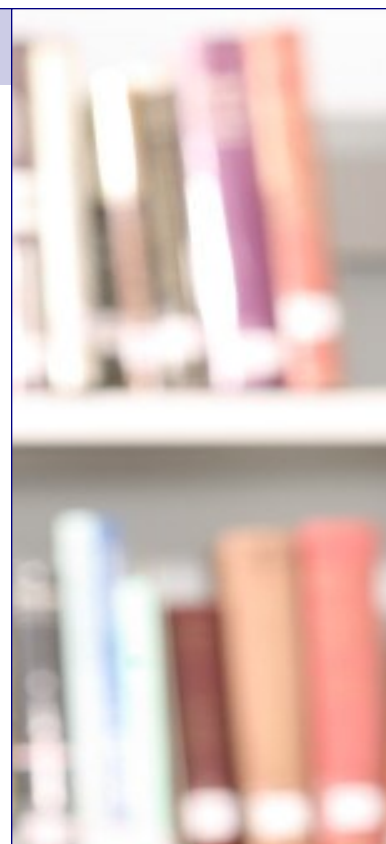


PAK Study Manual

Enterprise Risk Management (ERM) Exam
Spring 2022 Edition



PAK Study Manual for ERM Spring 2022

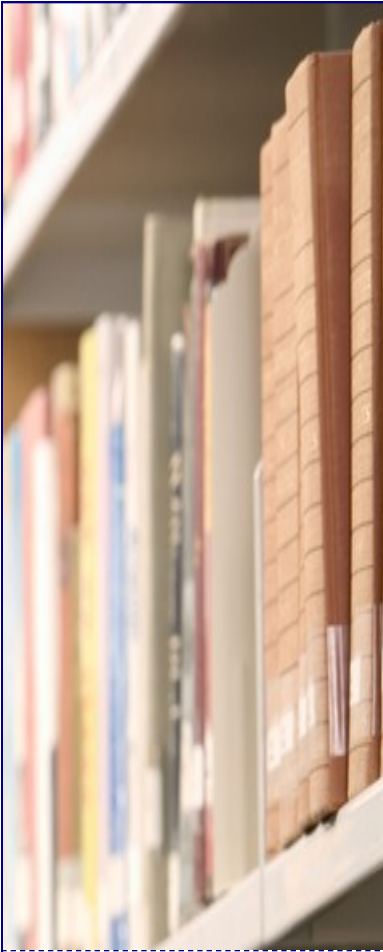


PRODUCT FEATURES

Features	PAK Study Manual	PAK Exam Aid	PAK Flash Cards	PAK Test Aid	PAK Online Seminar	PAK Study Manual Package
Summaries	X					X
Relevant Past Questions (List)	X					X
400+ Practice Questions	X					X
10 Mock Questions	X					X
Suggested Schedule	X					X
Spreadsheets (for Calculations)	X					X
Online Tutorial Videos	X					X
Email Support	X					X
110 Mock Questions		X				X
900+ Practice Questions		X				X
Case Study Analysis		X				X
Case Study Practice Question Set		X				X
Past Questions (Sorted PDFs)		X				X
Electronic Flash Cards			X			X
Audio + Anki Flash Cards			X			X
Condensed Summary			X			X
Mock Exam				X		X
Online Video Seminar					X	X
Practice Questions					X	X
Bonus materials						X

***All products are in electronic format.**

****The printed copy of the PAK Study Manual and the PAK Flash Cards can be purchased separately.**



PAK STUDY MANUAL

A printed copy of the manual can be purchased separately.

1. Summary (ERM Core + ERM Extension) (New)

The PAK Study Manual covers the entire Enterprise Risk Management (ERM) syllabus (***The ERM extension is included***). Not only does it give you the detailed explanations on conceptual, calculation, and exam materials, but it also fills in the gaps among the topics that are not covered in the source readings. It helps you better understand and master the confusing logics and difficult materials. In addition, it links the similar topics across readings together and connects them to the syllabus so that you can see the whole picture of this exam.

2. Relevant Past ERM SOA Exam Questions (List)

For each reading, we compose a list of relevant past exam questions (if any) so that you can locate the questions quickly and practice them immediately. This saves your time on searching what materials are relevant to this exam.

3. 400+ Practice Questions

One key point to pass this exam is to "practice" (Practice makes perfect!). Due to this reason, I include many practice questions in each reading (400+ in total) to refresh the materials just learnt and to strengthen your knowledge. More practice will be available in the PAK Exam Aid.

4. 10 Mock Exam Questions

The mock exam questions mimic the same difficulty level of the real exam questions. 10 mock exam questions and solutions are included in the PAK Study Manual to challenge your understandings. More practice will be available in the PAK Exam Aid.

5. Suggested Study Schedule (Detailed)

The syllabus is huge. It is very easy to lose track on your study. A clearly defined study schedule and some useful tips are included to help you better manage your schedule.

6. Spreadsheets (for Calculations) (New)

Useful spreadsheets are included to clarify the calculations in the syllabus.

7. Online Tutorial Videos

10 videos are included to cover the confusing and difficult topics.

8. Email Support

Get questions? Please send me an email .

"The study materials were very helpful in preparing me for the exam. Most importantly I was better able to apply the things that I learned to exam style questions. It was very organized and valuable." By Marc Roberts

[Read the whole story](#)

PAK FLASH CARDS

A printed copy of the flash cards can be purchased separately.

1. Electronic Flash Cards

- ◇ Summarize the key points in organized format.
- ◇ Include pros/cons, definition/description, etc.
- ◇ Contain around 350+ front/back flash cards (or 700+ slides).
- ◇ Read them in your smart phone device, tablet device, and/or computer.
- ◇ PDF version is also available.

2. Audio + Anki Flash Cards

- ◇ You can load them to your smartphone device and listen to them anytime/anywhere you want.
- ◇ An Anki version is available.

3. Electronic Condensed Summary

- ◇ Summarize the key points in outline format.
- ◇ Quickly refresh all the important topics in the readings.

PAK TEST AID

1. Mock Exam

- ◇ This set of mock exam is different from those mock questions available in the PAK Exam Aid. You can write down your answers and send them to me. I will give you detailed feedbacks on how to improve your exam score.

DO YOU KNOW?

The PAK Study Manual and related aids are updated EVERY exam sitting.

You will see the most updated materials, examples, and explanations to help you master the concepts and pass this exam in the first attempt.

PAK EXAM AID

1. 110 Mock Exam Questions and Solutions (with Case-Study-Related Questions and Spreadsheet-Based Calculations) (New)

The mock exam questions mimic the same difficulty level of the real exam questions. 100 mock exam questions and solutions (with **case-study-specific questions and spreadsheet-based calculations**) are included to challenge your understandings.

2. 900+ Practice Questions and Solutions (ERM Core + ERM Extension)

One key point to pass this exam is to "practice" (Practice makes perfect!). Due to this reason, I include many practice questions in each reading (900+ in total) to refresh the materials just learnt and to strengthen your knowledge. Please note that this practice question set is different from the practice question set in the manual.

3. Case Study Analysis (New)

This set connects the case study materials to the study materials so that you can see the picture on how they can be tested.

4. Case Study Practice Questions Set (New)

This set help students to better understand how to apply the knowledge into the case study.

5. Past SOA Exam Questions (from All FSA Tracks) Relevant to This Exam

This set not only includes the past exam questions from the ERM exam, but also includes the past exam questions from all the other FSA exam tracks (e.g. QFI, LP, IRM, etc). It helps you better understand how the materials were tested and gets you familiar with the SOA exam question style.

DO YOU KNOW?

You can find the most updated information about the PAK Study Manual and related aids under the "Announcement" section on the front page of the PAK website.

"The Mock questions, Mock exam and Exam Aid are also very beneficial to make sure you're abilities are up to par before the real exam." By Wes Smith

[Read the whole story](#)

RELEASE SCHEDULE

Features	PAK Study Manual	PAK Exam Aid	PAK Flash Cards	PAK Test Aid	PAK Online Seminar	PAK Study Manual Package
Summaries	11/15 + 12/15					11/15 + 12/15
Relevant Past Questions (List)	12/15					12/15
400+ Practice Questions	1/15					1/15
10 Mock Questions	12/15					12/15
Suggested Schedule	11/15					11/15
Spreadsheets (for Calculations)	12/15					12/15
Online Tutorial Videos	12/15					12/15
Email Support	Anytime					Anytime
110 Mock Questions*		2/15				2/15
900+ Practice Questions		2/15				2/15
Case Study Analysis*		2/15				2/15
Case Study Practice Question Set*		2/15				2/15
Past Questions (Sorted PDFs)		12/15				12/15
Electronic Flash Cards			12/15			12/15
Audio + Anki Flash Cards			2/15			2/15
Condensed Summary			12/15			12/15
Mock Exam*				3/15		3/15
Online Video Seminar					11/15 + 2/15	11/15 + 2/15
Practice Questions					2/15	2/15
Bonus materials						To be announced

* The release schedule of these items may be changed. It depends on when the SOA will release the new case study.



DO YOU KNOW?

If you are not sure which exam track to take, or how it can advance your career, you can send an email to Eddy and discuss your situation with him. He will share his work experience with you so that you can make your decision informatively.

PAK ONLINE SEMINAR

1. Over 70 video lectures to clarify and explain the key concepts/calculations in each syllabus reading (Core + Extension) and past exam questions
2. Analyze the new case study (e.g. how to answer the case-study-related questions)
3. Discuss the past exam questions (e.g. exam techniques, how to score)
4. Contain condensed outlines for each reading on the syllabus
5. Include practice questions (200+) to test your knowledge
6. Review the lectures and study at your own pace
7. On-demand video library compatible with smartphones (iPhone, Android, etc), tablets (iPad, Android), and PC/Mac devices.
8. Email support
9. **Free access for 2nd attempt**

PAK STUDY MANUAL PACKAGE

1. PAK Study Manual
2. PAK Exam Aid
3. PAK Flash Cards
4. PAK Test Aid
5. PAK Online Seminar
6. Bonus materials

SAMPLES?

You can find more samples on the [PAK](#) website.

IMPORTANT NOTES

1. Please note that all products (except flash cards) are in electronic (PDF) format. **No** hard copy is provided.
2. Once you make a purchase (please use your work email address), we will send you a confirmation email within 1 business days. Once the files are available, we will send them to you through email. Please make sure that you put the correct email address when you purchase the PAK products. **If you do not receive the confirmation email, please send us an email (services@pakstudymanual.com).**
3. Please check your "junk" mailbox. Sometimes, our email is blocked.

MORE INFORMATION

Want more information? Please contact me at eddy.chan@pakstudymanual.com or visit www.pakstudymanual.com

COMMENTS FROM THE PAST CANDIDATES

You can find more comments from the past candidates here: [PAK Testimonials](#).

WHERE TO PURCHASE PAK PRODUCTS

The PAK products are available at [Actex](#), and [Actuarial Bookstore](#).

Frequent Answer Questions

Do You Need to Read the Source Readings?

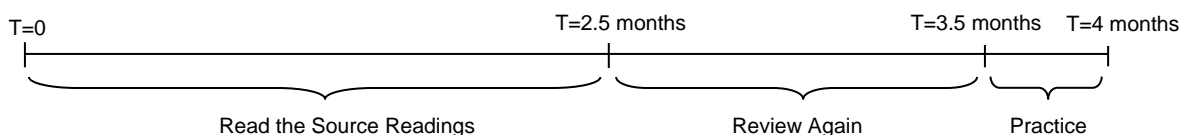
Unlike the preliminary exams, reading the source readings (textbooks, SOA study notes, and online readings) is **a must** in the FSA exams. PAK Study Manual can help you understand the materials faster and memorize them quickly so that in the time-limited environment, you can be well-prepared for the exam.

How Much Time is Needed to Study for This Exam?

This varies by person. Usually it will take one 300-350 hours to study for a FSA exam. Please expect to spend the same amount of time for the ERM exam.

Study Schedule

From the date the SOA release the new syllabus to the exam date, there are around 4 months to study. How to plan your study schedule?



Read the Source Readings

Assume you take the ERM exam in this exam sitting. In general, it will take one 2 to 2.5 months to finish the whole syllabus. To study more efficiently, we highly suggest you following the steps below:

Step 1: Define Your Own Study Schedule

- Use the suggested study schedule as a reference
- Prepare your own study schedule (*Target 20-30 pages @weekday and 50-60 pages @weekend*)
- Expect to read the whole syllabus and the past exams 2 or 3 times before the exam

Step 2: Read the Source Readings Together with the PAK Study Manual

- Write down your notes in the study manual
- Highlight all the key points there
- Label any calculations that you will go over again later
- Go over the related past exam questions once you finish that reading

Step 3: Practice the SOA Past Papers

- Practice them once you finish your first-round of readings (*use the PAK Exam Aid*)
- Understand how the topics were tested and how the questions were answered

Review Again

After completing the three steps above, you probably have a general idea about how the exam looks like. Now you should review the source readings again but this time focus more on the key topics, clarify the confusing concepts/calculations, think of what can be tested and read them carefully (use my mock exam questions)

Practice

The last month is the most critical month. Here are the steps:

- Practice the past exams and our mock questions to identify what you still do not know
- Go back to the readings and find your answers (*or send us an email if you need help*)
- Start memorizing the key points (*use the PAK Flash Cards*)
- Use the PAK Test Aid to test your knowledge (*Send us your answers and we will give you detailed feedbacks on how to improve your score in the exams*)

More Information

We will explain how to prepare for this exam in much more details in the PAK Study Manual.

ERM Extension

You will need to choose one of the six ERM extensions in the ERM exam. ***If you do not know which one fits you, please feel free to send me an email.***

Questions?

We know you probably have a lot of questions in your mind regarding the exam or choosing study aids. Please feel free to contact us at eddy.chan@pakstudymanual.com or paulpeterson@pakstudymanual.com.

Key Points**Key Points in This Reading**

- 1) Understand what the economic capital is and which types of risks are included in the economic capital
- 2) Understand how to calculate the economic capital with diversification benefits

Executive Summary

Two Meanings of Economic Capital

	Description
Required economic capital	○ It applies to the realistic amount a business believes it needs to meet future risks
Available economic capital	○ It indicates the realistic or market-consistent amount the business actually has available

Strike a Balance

- A company that properly employs economic capital analysis will strike a balance between:
 - Too much capital (can lead to an excessive cost of insurance)
 - Not enough capital (can lead to an unacceptable risk of insolvency)

1. IntroductionEconomic Capital

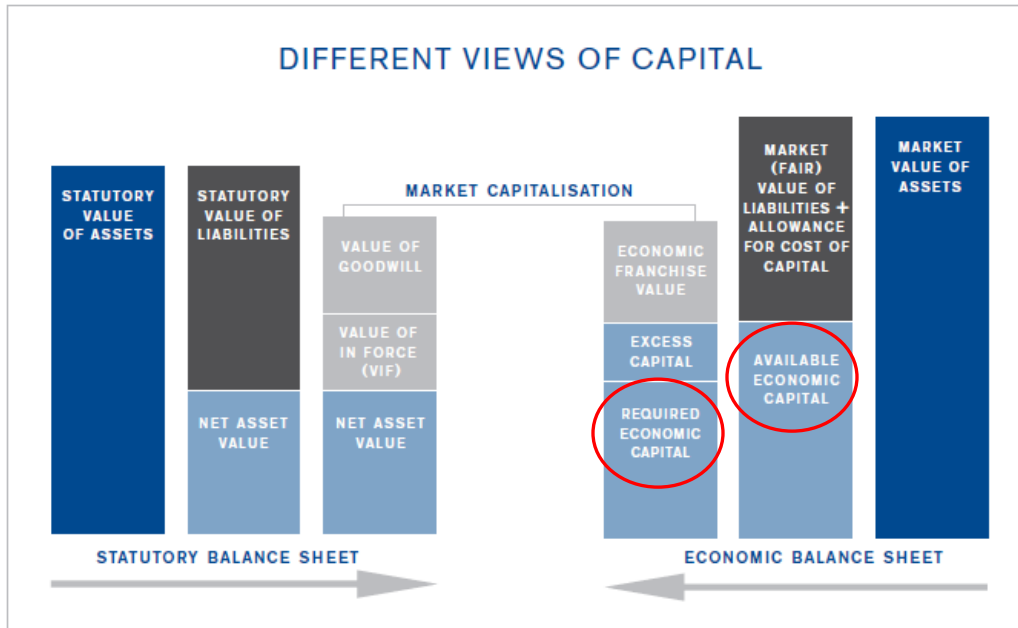
- Interpretation
 - The word “economic” is interpreted as referring to either a realistic or market-consistent valuation
 - The word “capital” refers to the discounted PV (capital) of future cash flows, or to the resources within the company’s balance sheet
- This concept can be used
 - To measure and optimize the capital resources already existing within a business, or
 - **To determine the amount of capital required** by a business to meet the risks inherent in its liabilities and business operations
- Two types of economic capitals
 - Required Economic Capital: The amount of economic capital a business believes it needs
 - Available Economic Capital: The amount of economic capital the business actually has
- Economic capital can be used
 - As part of the valuation of the business, or
 - As part of the risk management of the business
- More and more companies employ economic capital analysis
 - As it is recognized as a useful measure/tool for facilitating strategic management decisions
 - Due to regulatory pressure (Basel II, Solvency II, IAIS)

2. What is Economic Capital?

Memorization: Definition

Two Definitions of Economic Capital

	Description
Required economic capital	<ul style="list-style-type: none"> It is the capital required to support a business with a certain probability of default This capital is "required" from an economic point of view
Available economic capital	<ul style="list-style-type: none"> It is the excess of the value of the company's assets over the value of its liabilities on a realistic or market-consistent basis This definition is closely related to the European Embedded Value (EEV) standard



Different Views of Solvency

	Description
Statutory View of Solvency	<ul style="list-style-type: none"> Existing supervisory systems use simple formulae to define the required minimum level of solvency
Economic (Fair Value) View of Solvency	<ul style="list-style-type: none"> The market is moving towards "fair value" It equates to market-consistent valuation (on liabilities) The difference between the market value of assets and the market-consistent value of liabilities is the economic capital The required minimum level of solvency is defined as the required economic capital needed

3. What are the Benefits of Economic Capital Analysis?

Memorization: Benefits

Benefits of Economic Capital Analysis

- Determine the appropriate level of capital needed (*not too high* → costly; *not too low* → insolvency risk)
- Help management better understand the capital requirements and form discussion with rating agencies
- Measure performance based on capital usage
- Help shareholders assess the effective return on capital
- Help regulators understand how well capitalized companies are

4. How Can Economic Capital Analysis be Applied?

International Regulatory Trends

- Many regulators recognized many weaknesses in formulaic approaches to assess required solvency levels for insurance companies

Memorization: Weaknesses

Weaknesses in Formulaic Approaches

1. No link between the amount of required capital and the effectiveness of risk management
2. Cannot deal with all types of risks
3. Lack of transparency over the actual solvency level
4. Do not cope with changes in the financial/insurance markets
5. Do not allow for diversification benefit

Solvency II

- In 2000, the European Commission started the so-called Solvency II process
- It established the guidelines to develop more realistic solvency capital requirements for insurers

Two Levels of Solvency Capital

	Description
Solvency Capital Requirement (SCR)	<ul style="list-style-type: none"> o The SCR is the level of capital required so that there is a 0.5% probability that assets will not be sufficient to meet liabilities during the following year
Minimum Capital Requirement (MCR)	<ul style="list-style-type: none"> o The MCR is calculated based on a simple formula and represents an absolute minimum level of capital, below which urgent action would be required by the regulator

Internal Models

- Regulators permit insurers to calculate the SCR using internal models, provided these have been validated and approved by the regulator
- Solvency II is part of a convergence between economic and regulatory management of insurers
- Solvency II is expected to become effective around 2010

Memorization: Advantages

Advantages of Using an Internal Model to Calculate the SCR

1. Encourage insurers to measure and manage their risks
2. More flexible than industry-standard models
3. Can be updated as financial markets and a company's business evolve
4. Represent the insurer's business more closely than a rule-based standard approach

Risk-Based Capital (RBC)

- In the U.S., regulators use RBC as a measure of the sufficiency of surplus
 - o RBC is formulaic
 - o The use of internal capital models is adopted slowly
- In addition to using internal models and stochastics, RBC must be calculated using a deterministic projection set by the regulators (referred to as the Standard Scenario)
 - o The actual RBC is the greater of the amounts produced by the deterministic or stochastic projections

Canada

- Principles-based supervision and financial reporting have been in effect for several years
- Internal models and company-specific assumptions, with margins, are used

Swiss Solvency Test (SST)

- Switzerland uses the SST that is based on stochastic modeling and extreme scenarios
- It is formulated in terms of principles and guidelines defined by the supervisory authorities rather than by strict formulas

International Financial Reporting Standards (IFRS)

- It is also moving towards solvency-based regulation framework

European Embedded Value

	Description
Traditional embedded value approach <i>(previous version of EEV)</i>	<ul style="list-style-type: none"> o It allowed for the cost of holding the minimum regulatory capital o $PV(\text{Future capital release}) + PV(\text{after-tax investment income on capital}) - \text{Required capital at time 0}$ o But this did not allow for differences in the appropriate level of capital and the minimum regulatory capital
EEV	<ul style="list-style-type: none"> o It states that the level of required capital is at least the level of solvency capital required before the regulator can take action, but may also include amounts required internally o This leaves considerable discretion in determining the required capital for a particular company o Some companies have used economic capital as the basis for the cost of capital

Risk-Adjusted Return on Equity or Capital (RAROE or RAROC)

- Insurance products with different risks require different amounts of economic capital
- The return (embedded value profit) should be measured based on the economic capital employed
- RAROE or RAROC can be used to compare with the same measures employed in parallel businesses

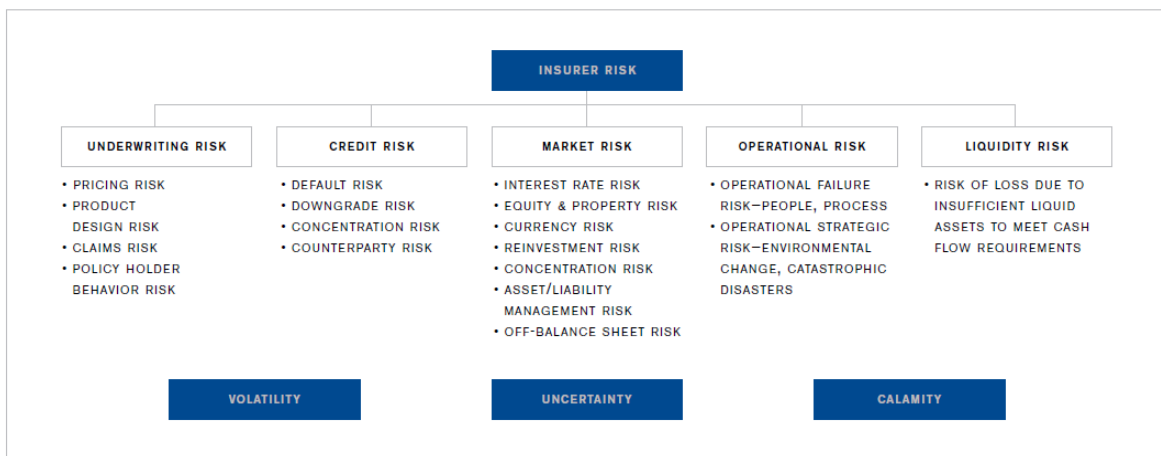
5. What Type of Risks Should be Considered?

Risks

- It is important to account for all risks that the insurer is exposed to for the purpose of EC calculation

Five Major Risk Types

	Description
Underwriting risk (insurance risk)	o Underwriting risks are associated with both the perils covered by the specific line of insurance and the specific processes associated with the conduct of the insurance business
Credit risk	o Credit risk is the risk of default and change in the credit quality of issuers of securities, counterparties, and intermediates to whom the company has an exposure
Market risk	o Market risk results from the volatility and uncertainty inherent in the market value of future cash flow from insurer assets and liabilities
Operational risk	<ul style="list-style-type: none"> o Operational risk is the risk of loss resulting from inadequate or failed internal process, people, or systems, or from external events o It is categorized by two components: operational failure risk and operational strategic risk
Liquidity risk	<ul style="list-style-type: none"> o Liquidity risk is exposed to loss in the event that insufficient liquid assets will be available to meet the cash flow requirements of the policyholder obligations when they are due or when assets may be available but only at excessive cost o Loss due to liquidity risk can occur when a company has to borrow unexpectedly or sell assets for an unanticipated low price



Three Key Components for Modeling

	Description
Volatility Risk	<ul style="list-style-type: none"> ○ Volatility is the risk of random fluctuations in either the frequency or severity of a contingent event <ul style="list-style-type: none"> ▪ In fully efficient markets, volatility is not market-valued since investors can reduce volatility by diversifying their portfolio ▪ However, due to the relatively inefficient markets for valuing insurance risks, the volatility component of risk cannot be ignored ○ E.g. mortality risk can be diversified by having a large number of policies <ul style="list-style-type: none"> ▪ That means smaller companies have more volatility risk than larger companies and thus should hold more capital per unit of insurance liabilities for a given probability of ruin
Uncertainty Risk	<ul style="list-style-type: none"> ○ Uncertainty involves the risk of misspecifying the model used to estimate the claims <ul style="list-style-type: none"> ▪ Uncertainty can also originate from misestimating the parameters within the models ▪ Uncertainty risk is nondiversifiable since it cannot be reduced by increasing portfolio size ○ E.g. term insurance has lesser uncertainty risk than medical insurance <ul style="list-style-type: none"> ▪ Since cost of medical insurance would depend not only on pure incidence rates but also on government policy, improvement of medical technology, economic downturn, and other social problems that are difficult to predict
Extreme Events (Calamity)	<ul style="list-style-type: none"> ○ They include the risk of : high-impact, low-frequency events (calamities) <ul style="list-style-type: none"> ▪ Models may not capture all aspects of extreme risk, especially if no extreme events appear in the historical data used to develop models ○ E.g. catastrophes with multiple claims, market crashes, or extreme interest rate movements

6. How Should Each of Those Risk Measured?Total Risk

- It is calculated by measuring the effect of specific (or aggregate) risk(s) to a company's earnings or surplus as a function of the probability distribution of losses

Several Approaches to the Measurement of Losses

	Description
Scenario-based model	<ul style="list-style-type: none"> ○ It can be deterministic or stochastic ○ Risk capital is calculated by measuring the impact of specific scenarios to the distribution of loss ○ These scenarios simultaneously cover multiple risk drivers
Static-factor model	<ul style="list-style-type: none"> ○ It is based on a linear combination of a static risk factor multiplied by a company-specific amount ○ RBC in the U.S. is an example of such an approach (except C3P1 and C3P2)
Stochastic-factor model	<ul style="list-style-type: none"> ○ A stochastic factor model is processed in the following steps: <ol style="list-style-type: none"> a. Identify relevant risk drivers b. Conduct a sensitivity analysis for each risk driver value to measure Delta, Gamma, or a scenario vector c. Model the joint distribution of risk drivers d. Aggregate the loss across all risk types and determine the risk capital as VaR or CTE
Covariance model	<ul style="list-style-type: none"> ○ A covariance model is a special case of a stochastic factor model with multi-normal distributions, first order sensitivities, and VAR as risk measure

(1) Underwriting Risk

- For life insurance, the major drivers of underwriting risk are
 - Mortality
 - Morbidity
 - Longevity
 - Lapse
 - Policyholder options

Diversifiable Component vs. Systematic Component

- Mortality, morbidity, and longevity risks can be further decomposed into 2 components:

	Description
A diversifiable component	<ul style="list-style-type: none"> ○ Diversifiable component (Volatility Risk) decreases as the number of policies increases ○ The volatility risk can be measured as the difference of liability amounts between best estimate and the Tail VaR under stochastic simulation of binomial mortality distribution
A systematic component	<ul style="list-style-type: none"> ○ It relates to mortality level risk (Misestimation of the Mean) and mortality trend risk (Deterioration of the Mean)

Mortality Level Risk

- The risk that actual mortality rates fluctuate around the mean due to the volatility of the historical data
 - A technique can be used to enhance the credibility of the estimate to certain confidence levels
 - The data is fit into a distribution and the distribution is used to develop the true mean

Mortality Trend Risk

- The risk that mortality trend differs from what is expected
- This may result from medical improvements or new diseases
- It is important, especially for products vulnerable to longevity risk

Catastrophic Mortality Risk

- It is not explicitly estimated since there are only limited experiences
- Life insurers reduce this risk through reinsurance agreements or issuing mortality bonds

Surrender and Lapse Risk

- It can be quantified in a similar way as described for the mortality risk

Risks Associated with a Policyholder's Options

- They depends on economic conditions and should be measured with stochastic analysis or a stress test
 - Once an interaction formula is defined, the risk can be quantified by conducting simulations under a number of economic scenarios
 - Judgments are needed since policyholder behavior is affected by factors other than economic conditions

(2) Credit Risk

- Credit risk is modeled in a way consistent with the banking standards.
 - Default, credit mitigation, spread, and spread volatility risks are considered
 - CreditRisk+, CreditMetrics, and KMV are industry-standard credit models in use

CreditRisk+

- It explicitly models the rates of default and recovery
 - It assumes that the default probability distribution follows a Poisson distribution with the parameters estimated by past data of default experience by credit class

Credit Metrics

- It is one of the credit migration models that consider the risk of default and the risk that an investment will lose or gain value due to changes in the corporation's credit rating

KMV

- It is the asset that models a firm's default as an option against its asset because the firm will go into default if the value of its assets becomes less than the value of its debt

Stochastic Factor Models

- They are used for modeling the reinsurance default risk
- The dependencies between reinsurance defaults, market risks, and catastrophic losses are considered

(3) Market Risk

- For insurers, market risk affects both assets and liabilities
 - o Changes in the MV of assets and liabilities
 - o Changes in the amount and/or timing of future liability cash flows
 - o Changes in policyholder behavior that impacts liabilities
- Due to the lack of real market for insurance liabilities, MV of liabilities needs to be technically derived
 - o Replicating portfolio can be used

(4) Operational Risk

- Operational risks (ORs) are either quantitatively modeled or qualitatively assessed, and may or may not lead to a capital charge

(A) Simple Add-On Model

- It aggregates OR by combining the anticipated costs for the various identified ORs
- Correlation and confidence level should be assumed

(B) Stochastic Frequency-Severity Model

- The main ORs are captured in each business unit through scenario analysis with experienced staffs and risk managers

7. What Modeling Decisions Should Inform the Analysis?**VaR vs. Tail-VaR**

	Description
VaR	<ul style="list-style-type: none"> o It assesses the probability of ruin at a given quantile of the probability distribution o It is commonly used in banking
Tail VaR	<ul style="list-style-type: none"> o It is defined as the arithmetic average of losses exceeding a given quantile o It considers both the probability and severity of losses that exceed a given quantile o It can deal better with low-frequency high-severity events because it takes more account of the shape of the tail of the distribution

Required Economic Capital

- It can be based on VaR or Tail VaR
- From a shareholder perspective
 - o VaR can be considered adequate because once the net worth has been exhausted, shareholders have lost the value of their shares and are not interested in the severity of further losses
- From a regulatory point of view
 - o The magnitude of losses is significant because it will determine the losses to policyholder and hence influence the damage to the reputation of the insurance industry and the regulator

Stochastic Analysis vs. Stress Test

- Both stochastic analysis and stress test are commonly used to see an impact of extreme events

Stochastic Analysis

- It can be defined as an analysis done by projecting future cash flow based on multiple scenarios of which **probability distribution is defined**
- In order to know the impact of extreme scenarios that would occur with x% probability, one needs to project cash flows under a large number of scenarios for the Monte Carlo simulation, but one needs just one scenario to know the probability distribution

Stress Test

- It can be defined as an analysis done by projecting future cash flow based on a (set of) particular scenario(s) that could occur in some extreme environments but for which **occurrence probability is not specified**
- The essential difference from stochastic analysis is that the occurrence probability of the stress scenario is not specified and it is not stochastically meaningful

Two Important Points When Calculating Economic Capital with a Certain Percentile of Confidence

- Stochastic analysis, where the occurrence probability is allocated to each scenario, is necessary to calculate economic capital, which is defined as an amount to cover future losses with a certain confidence level
- For risks for which it is difficult to determine the probability distribution, the occurrence probability is attached just for a technical purpose of economic capital calculation, and thus the user needs to understand its nature and the sensitivity to change of such subjective judgments

Real World Vv. Risk Neutral

Risk Neutral

- A risk-neutral technique is a method to calculate the PV of cash flows by discounting **risk-adjusted future cash flow** with **risk-free rates** based on multiple scenarios
 - o It is the consistent method to price derivative in the market
 - o It assumes no arbitrage and a complete market
 - o The risk-adjusted cash flow can be directly derived by utilizing the risk-free rates and implied volatilities available from the current market
 - o The risk-neutral technique is used because it is difficult to know the “adequate discount rates” under the P-measure

Real World

- A real-world technique is a method to calculate the present value of cash flows by discounting **projected cash flows** with **risk discount rates** based on multiple scenarios
 - o Projected cash flows are not adjusted for uncertainty risk
 - o To reflect the “price” of this uncertainty risk, the risk-discount rates are set higher than risk-free rates

The Risk-Neutral Technique

- It is superior in that the adjustment can be consistent with observable market prices of securities
- But there are two shortcomings during economic capital calculations:
 - o If economic capital is defined as a loss amount at 95%, the risk-neutral probability distribution should be converted so that the expected value ties to the 95 percentile but it is not straightforward how one can use the market price of securities to construct such a risk-neutral probability distribution
 - o Since the loss amount is derived as an expected value under a risk-neutral probability distribution instead of the 95 percentile under a real-world probability distribution, it is difficult to interpret the meaning of the results
- Because of these shortcomings, the real-world technique is widely used

Diversification Effect

- The total capital requirement could be less than the sum of the capital required for individual risks

Linear Correlation

- Linear correlation can be used to analyze risk dependencies
 - o But risk correlations can behave differently in extreme scenarios
 - o E.g. a big earthquake or terrorist attack could cause a mortality/morbidity surge and a huge drop in asset market prices simultaneously

Copulas

- This technique can be used to model dependency between risks
 - o A copula is a multivariate probability distribution function with uniform marginal distributions
 - o Although it can be applied to any kind of risk, copula functions are often used in conjunction with default risk modeling
- Difficulties when applying it to actual practices:
 - o There is no unique way to determine what kind of copulas should be used
 - o There are multiple methods to assess goodness of fit of copulas to sample data points

Diversification Credit

- Some parties (rating agencies) have been historically skeptical about giving full credit for diversification

Insurance Groups with Diverse Businesses

- They gain diversification in two ways
 - o The diversification effect of different independent risks
 - o The group diversification effect of non-correlated risks from their different businesses
- This diversification benefit helped to smooth earnings
- From a shareholder perspective
 - o This argument seems spurious because investors could achieve the same smoothing by diversifying their investment portfolios without having to invest in diversified businesses
 - o However, the benefits of diversification on capital management seem more concrete
- From the local regulators' perspective
 - o Diversification benefits from a group perspective may not be relevant since they need to ensure the solvency of each individual legal entities that they are supervising
 - o This has led the CRO Forum to propose that there should be a "Solo Entity Solvency Test" and a "Group Solvency Test"

Time Horizon to ConsiderTwo Methods in Measuring Risks and Required Capital in Internal Model

	Description
<i>A one-year model (covariance model)</i>	<ul style="list-style-type: none"> o The insurer could test the solvency for a short-term shock with a certain probability o The amount of capital required to survive the shock is the amount needed to ensure solvency o This method does not give information about the loss magnitude in the tail distribution o This approach is attractive to those companies wishing to avoid complex and time-consuming stochastic modeling since if solvency can be adequately estimated by using deterministic methods, then no stochastic modeling is required o Most regulators are in favor of adopting a one-year time horizon
<i>A multi-year / run-off model (stochastic scenario-based model)</i>	<ul style="list-style-type: none"> o The economic balance sheet of the company is projected for a long period (30 years) or until the liabilities have run-off o It can be structured such that there is adequate capital (1) throughout a certain percentage of these scenarios, or (2) only at the end of a certain percentage of these scenarios o A multi-year time horizon can give deeper understanding of the long-term risk exposures o But a possible weakness is that it may ignore management actions (hedging, capital raising)

Whether to Allow Negative Cumulative Surplus in the Middle of the Time Horizon

- More capital is needed if negative cumulative surplus is not allowed in the middle of a time horizon
- If it is allowed, one may want to assume some borrowings to cover such a shortfall, but it may introduce an issue of what borrowing rate one should use
- In practice, both methods are widely used

Whether to Account for Future New Business

- As a going-concern entity, it is important to confirm that writing new business does not jeopardize the company's economic capital
- Insurers should ensure that there is sufficient capital in each year after considering the impact of new business

8. Illustrative Examples

Example 1: Deterministic Stress Test

- This approach determines what the capital strain would be when an instantaneous shock to various risk factors is applied to the economic (or realistic) balance sheet
- By using an immediate stress, the approach simulates the impact on the economic balance sheet before management has time to react to it (so it does not include any allowance for management actions)
- This approach is a common way of determining capital requirements for companies in the UK

Data and Assumptions

- The stress test applies on a portfolio of unit-linked business with financial guarantees
- It assumes an instantaneous shock of -20% to equities, +5% nominal to volatility, and -1% to short-term interest rates

Starting Position

	(1)	(2) = (3) + (4)	(3)	(4)
Starting Position	Total Liabs	Total Assets	Equities	Bonds
Unit Reserve	100	100	50	50
Value of Guarantee	20	20	10	10
Capital	5	5	2.5	2.5
Total	125	125	62.5	62.5

Stress Scenario

	(1)	(2) = (3) + (4)	(3)	(4)
EQ?20%; i?1%; VOL?5%	Total Liabs	Total Assets	Equities	Bonds
Unit Reserve	2 90	90	1 40	50
Value of Guarantee	33.9	18	8	10
Capital	3 -11.4	4.5	2	2.5
Total	112.5	112.5	50	62.5

Stress Scenario

1. Balances for equities have all fallen by 20%
2. The value of guarantees has risen significantly from 20 to 33.9
3. The capital position has fallen from a small surplus of five to a significant deficit of -11.4
 - o The economic capital required (to absorb the decrease in surplus) = $-11.4 - 5 = 16.4$
 - o This amount would be sufficient to restore the balance sheet to an equivalent capital position

Hedging Position

	(1)	(2) = (3) + ... + (7)	(3)	(4)	(5)	(6)	(7)
Starting Position	Total Liabs	Total Assets	Equities	Bonds	Risk-Free Asset	Option	IR Swap
Unit Reserve	100	100	50	50			
Value of Guarantee	20	20	-23	-23	58	5	3
Capital	5	5	2.5	2.5			
Total	125	125	29.5	29.5	58	5	3

Hedging Position

- A dynamic hedging strategy is put in place to replicate the risks and value of the guarantee
 - o A short position in equities and bonds, combined with a long position in risk-free assets to hedge the market (delta) risk
 - o An equity option to hedge the volatility (vega) risk
 - o An interest rate swap to hedge interest rate (rho) risk

Stress Scenario

EQ?20%; i?1%; VOL?5%	(1)	(2) = (3) + ... + (7)	(3)	(4)	(5)	(6)	(7)
	Total Liabs	Total Assets	Equities	Bonds	Risk-Free Asset	Option	IR Swap
Unit Reserve	90	90	1 40	50			
Value of Guarantee	33.9	33	-18.4	-23	58	11.9	2 4.5
Capital	3 3.6	4.5	2	2.5			
Total	127.5	127.5	23.6	29.5	58	11.9	4.5

Stress Scenario

- All positions in equities have fallen by 20%
- The values of the option and interest rate swap have increased
- The impact of the stress test reduces the capital from 5 to 3.6
 - Thus the economic capital required is 1.4
 - This amount would be sufficient to restore the balance sheet to an equivalent capital position
 - By hedging the risks with derivative assets, the economic capital required to support this product has been significantly reduced from 16.4 to 1.4

Example 2: P&L (Profit and Loss) ProjectionThree Main Sources of Risk Underlying the Guarantee

	Description
Delta	<ul style="list-style-type: none"> Delta measures the rate of change of the value of the guarantee with respect to changes in the value of the underlying assets When underlying assets fall, the value of the guarantee increases significantly
Rho	<ul style="list-style-type: none"> Rho measures the rate of change of the value of the liability to changes in interest rates. If future interest rates fall, the risk-neutral PV of the liabilities increases, due to the lower expected risk-neutral growth rate of the underlying assets and the lower interest rate used to convert future guarantee claims to present values
Vega	<ul style="list-style-type: none"> Vega measures the rate of change of the value of the liability to changes in volatility The greater the volatility, the greater the cost of hedging and the greater the value of the guarantee liabilities

Financial Projection of the P&L

- The guarantee and the hedging derivative portfolio are projected
- The graphs show before and after including the hedges
- The economic capital is defined as VaR and CTE at 95% and 99% level

Hedged Position

- The reduction in risk as measured by the volatility or dispersion of P&L results is significant
- There is a significant reduction in the economic capital required to support the business when the effect of hedging has been taken into account

Conclusion of this Example

- Most risk management strategies do not normally eliminate all risks completely
- Rather, they reduce the amount of risk down to much lower levels than would have been the case had the risk management strategies not been in place

Example 3: Holistic VAR Aggregation — Towards Enterprise Risk Management (ERM)

- This example considers each source of risk individually and aggregates them using a correlation matrix

Risk Sources

Risk Sources	Description
Market Risk	○ The risk that market levels, interest rates, or volatility change, resulting in losses
Credit Risk	○ The risk that an asset held experiences either a default or a significant fall in its credit quality (an increase in its credit spread over risk-free bonds)
Liquidity Risk	○ The risk that reduced liquidity constrains the ability to buy and sell assets, resulting in losses
Underwriting/Demographic Risk	○ The risk that actual demographic experience turns out worse than expected
Mortality/Longevity/Morbidity Risk (Diversifiable component)	○ It reduces as the number of insured lives increases due to the central limit theorem
Mortality/Longevity/Morbidity Risk (Systematic component)	○ It relates to the risk that the mortality basis is incorrect
Lapse Risk	○ The risk that policyholders will lapse differently from that expected
Policyholder Behavior Risk	○ The risk that policyholders will elect various options (annuitization rates, withdrawal rates, etc.) differently from that expected
Expense Risk	○ The risk that experienced expenses are higher than those assumed when pricing a product at the outset
Operational Risk	○ The risk that there is a failure in the operational aspects of the business—people, processes, or systems
Group Risk	○ The risk that a loss or failure of a related group entity results in a loss of the entity under consideration

Correlation Matrix

MILLIONS	MORTALITY	LONGEVITY	MORBIDITY	LAPSE	EXPENSE	
MORTALITY	1.0	0.0	0.5	0.0	0.5	
LONGEVITY	0.0	1.0	0.0	0.5	0.5	
MORBIDITY	0.5	0.0	1.0	0.0	0.5	
LAPSE	0.0	0.5	0.0	1.0	0.5	
EXPENSE	0.5	0.5	0.5	0.5	1.0	
ECONOMIC CAPITAL	9.0	13.0	5.0	35.0	15.0	
ECONOMIC CAPITAL WITHOUT DIVERSIFICATION						77.0
ECONOMIC CAPITAL WITH DIVERSIFICATION						56.0

Aggregation Examples

Economic capital without diversification (assume all risks are perfectly correlated)

$$= 9 + 13 + 5 + 35 + 15$$

$$= 77$$

Economic capital with diversification

$$= \left(\begin{bmatrix} 1 & 0 & 0.5 & 0 & 0.5 \\ 0 & 1 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 1 & 0 & 0.5 \\ 0 & 0.5 & 0 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 13 \\ 5 \\ 35 \\ 15 \end{bmatrix} \right)^{0.5}$$

$$= \left(\begin{bmatrix} 9 \times 1 + 13 \times 0 + 5 \times 0.5 + 35 \times 0 + 15 \times 0.5, \dots, 9 \times 0.5 + 13 \times 0.5 + 5 \times 0.5 + 35 \times 0.5 + 15 \times 1 \end{bmatrix} \begin{bmatrix} 9 \\ 13 \\ 5 \\ 35 \\ 15 \end{bmatrix} \right)^{0.5}$$

$$= \left(\begin{bmatrix} 19 & 38 & 17 & 49 & 46 \end{bmatrix} \begin{bmatrix} 9 \\ 13 \\ 5 \\ 35 \\ 15 \end{bmatrix} \right)^{0.5}$$

$$= (19 \times 9 + 38 \times 13 + 17 \times 5 + 49 \times 35 + 46 \times 15)^{0.5}$$

$$= 56$$

Correlation Matrix

- It allows for the fact that the various risk factors are not perfectly correlated
- This results in a reduction in the economic capital required from 77 to 56 million

Aggregation in Source Risk Level

	Market	Credit	Liquidity	Insurance	Operational	Group	Aggregate
Expected	1000	1000	1000	1000	1000	1000	6000
5%	1203	1173	1183	1056	1356	1215	7186
5% VaR	203	173	183	56	356	215	1186
5% VaR with Diversification							816

VaR Calculation

$$x\% \text{ VaR} = x\text{th percentile} - \text{Expected value}$$

$$5\% \text{ VaR}_{\text{Market}} = 1203 - 1000 = 203$$

Economic Capital

- Without diversification, it is \$1,186
- With diversification, it is 816 (a reduction of 31%)
- This is the economic capital needed for the UL Guarantee product

Total Economic Capital for the Whole Company

- Summing across all products and risks gives us an estimate of 1,985 for the economic capital assuming perfect correlation between risks
- When diversification benefits are taken into account, this reduces to 1,588

Appendix: Examples of the IAA Classification of Insurer Risk**(1) Underwriting Risk**

	Description
Underwriting process risk	o Risk from exposure to financial losses related to the selection of risks to be insured
Pricing risk	o Risk that the prices charged by the company for insurance contracts will be ultimately inadequate to support the future obligations arising from those contracts
Product design risk	o Risk that the company faces risk exposure under its insurance contracts that were unanticipated in the design and pricing of the insurance contract
Claims risk	o Risk that many more claims occur than expected or that some claims that occur are much larger than expected claims resulting in unexpected losses
Economic environment Risk	o Risk that social conditions will change in a manner that has an adverse effect on the company
Net retention risk	o Risk that higher retention of insurance loss exposures results in losses due to catastrophic or concentrated claims experience
Policyholder behavior risk	o Risk that the insurance company's policyholders will act in ways that are unanticipated and have an adverse effect on the company
Reserve risk	o Risk that the provisions held in the insurer's financial statements for its policyholder obligations will prove to be inadequate

(2) Credit Risk

	Description
Direct default risk	o Risk that a firm will not receive the cash flows or assets to which it is entitled because a party with which the firm has a bilateral contract defaults on one or more obligations
Downgrade or migration risk	o Risk that changes in the possibility of a future default by an obligor will adversely affect the present value of the contract with the obligator today
Indirect credit or spread risk	o Risk due to market perception of increased risk
Settlement risk	o Risk arising from the lag between the value and settlement dates of securities transactions
Sovereign risk	o Risk of exposure to losses due to the decreasing value of foreign assets or increasing value of obligations denominated in foreign currencies
Concentration risk	o Risk of increased exposure to losses due to concentration of investment in a geographical area or other economic sector
Counterparty risk	o Risk of changes in values of reinsurance, contingent assets, and liabilities

(3) Market Risk

	Description
Interest rate risk	o Risk of exposure to losses resulting from fluctuations in interest rates
Equity and property risk	o Risk of exposure to losses resulting from fluctuation of market values of equities and other assets
Currency risk	o Risk that the relative changes in currency values decrease value of foreign assets or increase the value of obligations denominated in foreign currencies
Basis risk	o Risk that yields on instrument of varying credit quality, liquidity, and maturity do not move together, thus exposing the company to market value variation that is independent of liability values
Reinvestment risk	o Risk that the returns on funds to be reinvested will fall below anticipated levels
Concentration risk	o Risk of increased exposure to losses due to concentration of investment in a geographical area or other economic sector
Asset/liability mismatch risk	o Risk in changes to capital levels to the extent that the timing of amount of the cash flows from the assets supporting the liabilities and the liability cash flows are different
Off-balance sheet risk	o Risk of changes in values of contingent assets and liabilities such as swaps that are not otherwise reflected in the balance sheet

Practice Question

Given the data on p.43 and p.44, calculate the 5% VaR with diversification

	Market	Credit	Liquidity	Insurance	Operational	Group	Aggregate
5%	1203	1173	1183	1056	1356	1215	7186
5% VaR	203	173	183	56	356	215	1186

	Credit	Liquidity	Market	Operational	Insurance	Group
Credit	1.00	0.30	0.40	0.20	0.25	0.10
Liquidity	0.30	1.00	0.50	0.25	0.10	1.00
Market	0.40	0.50	1.00	0.10	0.10	0.50
Operational	0.20	0.25	0.10	1.00	0.10	0.50
Insurance	0.25	0.10	0.10	0.10	1.00	0.40
Group	0.10	1.00	0.50	0.50	0.40	1.00

5% VaR with diversification

$$\begin{aligned}
 &= \left(\begin{matrix} & \begin{matrix} 1 & 0.3 & 0.4 & 0.2 & 0.25 & 0.1 \end{matrix} \\ \begin{matrix} 173 & 183 & 203 & 356 & 56 & 215 \end{matrix} & \begin{matrix} \begin{bmatrix} 173 \\ 183 \\ 203 \\ 356 \\ 56 \\ 215 \end{bmatrix} \end{matrix} \end{matrix} \right)^{0.5} \\
 &= \left(\begin{matrix} & \begin{matrix} 173 \\ 183 \\ 203 \\ 356 \\ 56 \\ 215 \end{matrix} \end{matrix} \right)^{0.5} \\
 &= \left(\begin{matrix} & \begin{matrix} 173 \\ 183 \\ 203 \\ 356 \\ 56 \\ 215 \end{matrix} \end{matrix} \right)^{0.5} \\
 &= 816
 \end{aligned}$$

Useful Concept

Question #1: One of the weaknesses in the formulaic approaches is that it does not allow for diversification benefit. However, I thought RBC was a formulaic approach, and it does account for diversification due to the square root of the squared components. So why does it state the formula approach does not allow for diversification benefit?

Answer #1: In a sense, it does, but only among certain sets of risks in certain sets of products. For example, if you have annuities and life insurance in the company, RBC formulas might not take that into account (since the square root is only around the specific factors for each business). Also, the square root assumes only 0 correlation, even if we actually had negative correlation.

There are some covariance adjustments that can be made, but they are modest for the most part, and we end up holding more capital than if we had "true" diversification.

Question #2: In addition to the formulaic approach for RBC, they must also calculate capital using deterministic projects set by regulators. It also mentions that they use stochastic projections. How does this work? Also, is Solvency 2 calculated entirely using stochastic models?

Answer #2: This comes up a lot in Variable Annuities. For a piece of the RBC formula, we have to model out interest and equity rate risks. We do this two ways: deterministically (the Standard Scenario) and stochastically, then take the greater of the two. This, then, ends up being one of the factors in the RBC.

Solvency II is mostly stochastic based, with some deterministic floors, as I understand it. But they do put a MUCH greater emphasis on stochastics than RBC.

Question #3: The scenario-based model can be deterministic or stochastic. If it stochastic, is it the same as the stochastic-factor model? What is the difference between the two?

Answer #3: Yes, this statement is a bit weird. To be honest, scenario-based models are, almost by definition, deterministic, because we are trying to look at a certain scenario. My guess is that they have some deterministic scenario they are modeling, but then through in some stochastic variables (i.e., deterministic equity returns with stochastic mortality).

Question #4: When discussing the different modeling decisions which should inform the analysis, it is mentioned that real world techniques are better for determining economic capital than risk-neutral techniques. What is the reason?

Answer #4: Risk-neutral assume that all assets (stocks bonds, etc.) earn the risk-free rate on an expected value basis. This works nicely when we are pricing securities, like calls and puts, but when we try to project using this method, things look weird. For example, do we really think the S&P 500 will have 10yr return equal to the return on a 10yr Treasury? Do we think 10yr Treasury's and junk bonds will return the same? Probably not.

So then we move to the real-world view, which allows for risk premiums. This shows us better what might "actually" happen, which is better for our risk management purposes.

Risk-neutral is good for pricing things *today*. Real-world is good for projecting things *tomorrow (future)*.

SOA ERM Past Exam Questions Related To This Reading

SOA ERM Spring 2020 Q4c	(Must Read)
SOA ERM Fall 2017 Q1b	(Must Read)
SOA ERM Spring 2017 Q5	(Must Read)
SOA ERM Spring 2016 Q1, 5a-b	(Must Read)
SOA ERM Spring 2013 Q4	(Must Read)

PAK Study Manual (Practice Questions)
for ERM Spring 2022
(Sample)

Note

1. 400+ Practice Questions/Solutions are included in the PAK Study Manual.

Textbook Reading: Value at Risk Ch.12

Q1: Securities firms commonly use simulation techniques (known as Monte Carlo methods) to value complex derivatives and to measure risk. Simulation methods approximate the behavior of financial prices by using computer simulations to generate random price paths. Compare the pros and cons of using Monte Carlo simulation.

Q2: Suppose each interval is one day. The initial stock price is \$60. The instantaneous drift rate is 0.6% and the instantaneous volatility is 3%. Assume we generate two standard normal random variable: the first random variable is -0.80, and the second one is 0.90. The first random variable will be used to calculate the ending stock price at day 1. The second variable will be used to calculate the ending stock price at day 2. Calculate the ending stock price at day 2.

Q3: Bootstrap method is used to generate random numbers by sampling from historical data (empirical distribution). Compare the pros and cons of using bootstrap method.

$$\text{Covariance Matrix} = \begin{bmatrix} 0.04 & 0.16 & 0.10 \\ 0.16 & 0.73 & 0.46 \\ 0.10 & 0.46 & 0.54 \end{bmatrix}$$

Q4: Suppose we have a covariance matrix R above. It can be decomposed into its Cholesky factors: $R = TT'$, where T is a lower triangular matrix with zeros in the upper right corners. Calculate the T .

Q5-Others are not shown in this sample.

Textbook Reading: Value at Risk Ch.12**S1:**

Pros and Cons of Using Monte Carlo simulation

Advantages	Disadvantages
1. It is the most powerful approach to VAR due to its flexibility	1. This approach involves costly investments in intellectual and systems development
2. It accounts for a wide range of risks and complex interactions	2. It requires substantially more computing power than simpler methods
3. It accounts for nonlinear exposures and complex pricing patterns	
4. Simulations can be extended to longer horizons	
5. It can be used for operational risk measurement, and integrated risk management	

S2:

Step i	Initial Price S_{t+i-1}	Random Variable ε_i	Increment $\Delta S = S_t (\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t})$	Current Price $S_{t+i} = S_{t+i-1} + \Delta S_{t+i-1}$
1	60.00	-0.80	-1.08	58.92
2	58.92	0.90	1.94	60.86

$$\Delta S_t = S_t (\mu\Delta t + \sigma\varepsilon\sqrt{\Delta t})$$

$$S_{t+1} = S_t + \Delta S_t$$

$$\Delta S_1 = 60 \times (0.006 \times 1 + 0.03 \times (-0.80) \times \sqrt{1}) = -1.08$$

$$S_1 = 60 + (-1.08) = 58.92$$

$$\Delta S_2 = 58.92 \times (0.006 \times 1 + 0.03 \times (0.90) \times \sqrt{1}) = 1.94$$

$$S_2 = 58.92 + 1.94 = 60.86$$

S3:

Pros and Cons of Using Bootstrap Method

Advantages	Disadvantages
1. It can include fat tails jumps, or any departure from the normal distribution	1. For small sample sizes M , the bootstrapped distribution may be a poor approximation of the actual one so sufficient data points are needed
2. It accounts for correlations across series because one draw consists of the simultaneous returns for N series	2. It relies heavily on the assumption that returns are independent so by resampling at random, any pattern of time variation is broken

Q4-Others are not shown in this sample.